

# Homework 4–21-241, Matrices and Linear Transformations

Name: \_\_\_\_\_  
Section: \_\_\_\_\_

**Instructions:** Complete the following problems, clearly labeling the problems. Staple this sheet, with your name and section filled in, to the top of your work. Failure to attach this sheet will result in a one point deduction in the grade. The assignment will be graded out of ten points.

**DUE: Friday, February 24, 2017**

## Book Problems

1. Section 3.5: 2, 4, 6, 12, 16, 18, 28, 34, 40, 42, 52, 58
2. Section 3.6: 4, 6, 10, 14, 16, 18

## Other Problems

1. Let  $A$  be an  $m \times n$  matrix. Show that for any non-zero  $\mathbf{b} \in \mathbb{R}^m$  such that  $A\mathbf{x} = \mathbf{b}$  is consistent, the set  $S := \{\mathbf{x} \in \mathbb{R}^n \mid A\mathbf{x} = \mathbf{b}\}$  fails to meet any of the three requirements from the definition of subspace given in class (or the definition on page 192 in your book).
2. Let  $S$  be a subspace of  $\mathbb{R}^n$  and let  $B = \{\mathbf{v}_i\}_{i=1}^n$  be a subset of  $S$  such that each vector in  $S$  can be written uniquely as a linear combination of the vectors in  $B$ . Prove that  $B$  is a basis for  $S$ . (We proved the other direction of this implication in class, so in fact,  $B$  being a basis of  $S$  is equivalent to each vector in  $S$  being expressed uniquely as a linear combination of the vectors in  $B$ .)

3. Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 & -4 \\ 1 & 0 & 0.25 & -1 \\ -2 & -1.1 & 0.25 & 6.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix}.$$

- (a) Use MATLAB to find an  $LU$  factorization of  $A$  (see page 38 of Appendix E).
- (b) Let

$$\mathbf{b} = \begin{bmatrix} -3 \\ -1.5 \\ 5.6 \\ 2.2 \end{bmatrix}.$$

Use the  $LU$  factorization of  $A$  to solve  $A\mathbf{x} = \mathbf{b}$ .