

Answer the questions below. You may answer in the space provided. You may use the back or a separate sheet of paper if you need more space. You are to work in groups of no more than four people. Make sure to enter the names of your groupmates below.

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Group Members: \_\_\_\_\_

1. (10 points) Let  $A$  be the matrix  $\begin{bmatrix} 3 & -5 \\ 1 & -3 \end{bmatrix}$ . Given that the eigenvalues of  $A$  are  $-2$  and  $2$  with corresponding eigenvectors  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$ , respectively, calculate  $A^9$  by finding a diagonal matrix  $D$  similar to  $A$ , that is,  $D = P^{-1}AP$  for some invertible matrix  $P$ , and use that  $A^9 = PD^9P^{-1}$ .

2. (5 points) Show that if  $A$  and  $B$  are invertible  $n \times n$  matrices, then  $AB$  and  $BA$  have the same eigenvalues.  
**Hint:** Show  $AB \sim BA$ .

3. (5 points) Let  $\lambda$  be an eigenvalue of  $A$  with corresponding eigenvector  $\mathbf{x}$ . Show that  $\lambda + t$  is an eigenvalue of  $A + t\mathbb{I}$  with corresponding eigenvector  $\mathbf{x}$ .