Answer the questions below. You may answer in the space provided. You may use a separate sheet of paper if you need more space. You are to work in groups of no more than four people. Make sure to enter the names of your groupmates below.

Name: ______________________________

Section: ______________________________

Group Members: ______________________________

1. Determine if the matrices

$$\begin{bmatrix} 0 & 1 \\ 5 & 2 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} -2 & -1 \\ 0 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} -1 & -3 \\ 1 & 9 \\ 4 & 5 \end{bmatrix}$$

are linearly independent.

(a) (4 points) What is the homogeneous system of linear equations you should consider to determine if the above matrices are linearly independent?

(b) (4 points) Solve this system and use the solution to determine if the matrices are linearly independent. You may use a calculator to find a row echelon form for the augmented matrix for the system above.

2. (6 points) Using the properties of matrix multiplication, show that for square matrices of the same size A and B, $AB = BA$ if and only if $(A-B)(A+B) = A^2 - B^2$ (remember, in general, matrix multiplication is not commutative).

3. (6 points) The trace of a square, $n \times n$ matrix A is defined as the sum of the entries on the main diagonal, that is

$$\mathrm{tr}(A) := \sum_{i=1}^{n} a_{ii}.$$

Prove that for $n \times n$ matrices A and B, $\mathrm{tr}(A+B) = \mathrm{tr}(A) + \mathrm{tr}(B)$.