Answer the questions below. You may answer in the space provided. You may use the back or a separate sheet of paper if you need more space. You are to work in groups of no more than four people. Make sure to enter the names of your groupmates below.

Name: ______________________________

Section: ______________________________

Group Members: ______________________________

On this assignment, you will practice some proofs, which will be a needed skill set for this course.

1. (4 points) Prove directly: Let $\mathbf{u}$, $\mathbf{v} \in \mathbb{R}^n$. Then $\|\mathbf{u}+\mathbf{v}\|^2 + \|\mathbf{u}-\mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2$.

2. (6 points) Prove that there are no vectors $\mathbf{u}$, $\mathbf{v} \in \mathbb{R}^n$ such that $\|\mathbf{u}\| = 5$, $\|\mathbf{v}\| = 2$, and $\mathbf{u} \cdot \mathbf{v} = 20$.

3. This question involves a proof by induction. To prove by induction, we prove the statement on some base case (typically for $n = 0$ or $n = 1$, but for this problem, the base case will be for $n = 2$) and then show that by assuming the statement is true for $n = k$, it must also be true for $n = k + 1$. By proving these two things, we then know that the statement is true for any natural number n. Prove the following by induction on n: For any $n \in \mathbb{N}$ and $\mathbf{v}_i \in \mathbb{R}^m$, $i = 1, \ldots, n$, then

$$\|\mathbf{v}_1 + \mathbf{v}_2 + \cdots + \mathbf{v}_n\| \leq \|\mathbf{v}_1\| + \|\mathbf{v}_2\| + \cdots \|\mathbf{v}_n\|.$$

   (a) (2 points) Prove the statement for $n = 1$

   (b) (2 points) Prove the statement for $n = 2$.

   (c) (6 points) Assume the statement holds for $n = k$. Prove that it holds for $n = k + 1$.