Answer the questions below, showing your work justifying your answers. You may answer in the space provided. You may use a separate sheet of paper if you need more space. You are to work in groups of no more than four people. Make sure to enter the names of your groupmates below.

Name:
Section: $\qquad$

Group Members: $\qquad$

1. Consider the equation

$$
\begin{equation*}
A x=b \tag{1}
\end{equation*}
$$

for an unknown real number $x$ and given real numbers $A$ and $b$. Answer the following questions, explaining how you obtained your answers.
(a) (3 points) Under what conditions on $A$ and $b$ does (1) have exactly one solution? What is this solution?
(b) (3 points) Under what conditions on $A$ and $b$ does (1) have infinitely many solutions? What values of $x$ can be a solution?
(c) (2 points) Under what conditions on $A$ and $b$ does (1) have no solution?
2. You have probably learned two ways to solve systems of two linear equations in two unknowns: substitution and elimination (although you may not have used those terms). In substitution, you solve one of the equations for one of the variables, substitute this into the other equation and use the result to solve for the other variable, and then the variable you substituted.

In elimination, you multiply one or both of the equations by a non-zero constant(s) so that when you add the two new equations, one of the variables cancels, enabling you to solve for the other. You then use this result to solve for the second variable.
(a) (5 points) Using the method of substitution, solve the system

$$
\left\{\begin{array}{l}
x+y=5 \\
3 x+3 y=10
\end{array}\right.
$$

(b) (5 points) Using the method of elimination, solve the system

$$
\left\{\begin{array}{l}
2 x-y=5 \\
3 x+4 y=2
\end{array}\right.
$$

