21-241 Lec5 Exam 4 Study Guide

If you understand and can do the following things, you should do well on the fourth exam. Remember, it is closed book, closed notes, no electronic devices except internet-less calculators, etc. You should also be comfortable writing mathematical proofs. On the exam, your proofs should meet the following specifications:

1. everything stated should be true and explained where appropriate (Yes, I know this is vague, but sometimes this is a stylistic decision. When in doubt, explain why your statement is true.),
2. the proof needs to prove the actual statement in question, and
3. the proof needs to be written in clear mathematical English (that is, there should be words explaining what you are doing).

1 What You Need to Know

You should be comfortable with/know/be able to do

- span and linear independence of a set of vectors in a vector space
- the definition of basis of a vector space
- determining if a set is a basis
- coordinate vectors with respect to a basis
- the Dimension Theorem and its consequences
- extending a set of vectors to a basis
- change of basis matrices, their properties, and how to calculate them
- linear transformations between vector spaces, how to prove a given function is a linear transformation, and their properties
- injectivity, surjectivity, and bijectivity
- inverses of functions/linear transformations
- kernel and range of a linear transformation

- the Rank Theorem and its consequences
- isomorphisms and how to prove a mapping is an isomorphism
- showing spaces are isomorphic
- matrices of linear transformations
- inner products and inner product spaces
- orthogonal decomposition for general inner product spaces
- Gram-Schmidt for inner product spaces
- Cauchy-Schwarz inequality for inner product spaces.

2 Note About Using Calculators

As I have said in class, you are allowed to use graphing calculators, so long as they do not have the capability of connecting to any computer network. As such, it is not expected on the exam that you do row reductions by hand or calculate inverses of matrices by hand (granted, this second one is a case of row reduction). When you are showing your work on the exam, it is sufficient to make clear which matrix you are putting into row echelon form or reduced row echelon form (indicating which one of the two), and then giving the reduced form, labeling it as such. For example, if you need to find the reduced row echelon form of the matrix A, for whatever reason, you may write

$$\text{rref}(A) = \cdots$$

where $\cdots$ is the reduced row echelon form of the matrix. When calculating the determinant of a matrix, you should indicate the cofactor expansion you are using to reduce the problem to finding determinants of 2×2 matrices.