21-241 Lec5 Exam 3 Study Guide

If you understand and can do the following things, you should do well on the third exam. Remember, it is closed book, closed notes, no electronic devices except internet-less calculators, etc. You should also be comfortable writing mathematical proofs. On the exam, your proofs should meet the following specifications:

1. everything stated should be true and explained where appropriate (Yes, I know this is vague, but sometimes this is a stylistic decision. When in doubt, explain why your statement is true.),
2. the proof needs to prove the actual statement in question, and
3. the proof needs to be written in clear mathematical English (that is, there should be words explaining what you are doing).

1 Similarity

You should know/be able to do

- show two matrices are similar,
- know what properties two similar matrices share
- diagonalize a matrix, if the matrix is diagonalizable,
- know necessary and sufficient conditions for a matrix to be diagonalizable.

2 Orthognality

You need to be comfortable with/comfortable with using

- showing that a set of vectors in $\mathbb{R}^n$ is orthogonal/orthonormal,
- easily determining the coordinates of a vector in $\mathbb{R}^n$ with respect to an orthogonal basis,
- orthogonal matrices and their properties,
- orthogonal complements,

- the four fundamental spaces of a matrix and how they are orthogonally related,
- orthogonal projections and decompositions,
- using the Gram-Schmidt process to find an orthogonal/orthonormal basis of a subspace of $\mathbb{R}^n$ given a basis, or vectors to be in the basis of the subspace,
- QR factorization,
- orthogonal diagonalization (how to do it, how to know when it is possible for real matrices)
- the Spectral Theorem (how to use it, not prove it).

3 General Vector Spaces

You need to be comfortable with/comfortable with using

- the definition of a vector space over general scalar fields (usually $\mathbb{R}$ or $\mathbb{C}$),
- proving a set with a field of scalars and given addition and scalar multiplication is a vector space or not,
- subspaces of vector spaces,
- linear combinations and spanning in arbitrary vector spaces,
- linear independence/dependence and bases in vector spaces
- coordinate vectors.

You may assume on the exam that $\mathbb{R}^n$, $\mathbb{C}^n$, the set of $m \times n$ matrices M_{mn}, the set of polynomials $\mathcal{P}$, the sets of polynomials $\mathcal{P}_n$ of degree at most n, and the space $\mathcal{F}$ of functions from $\mathbb{R}$ to $\mathbb{R}$ are vector spaces with their usual additions and scalar multiplications.

4 Note About Using Calculators

As I have said in class, you are allowed to use graphing calculators, so long as they do not have the capability of connecting to any computer network. As such, it is not expected on the exam that you do row reductions by hand or calculate inverses of matrices by hand (granted, this second one is a case of row reduction). When you are showing your work on the exam, it is sufficient to make clear which matrix you are putting into row echelon form or reduced row echelon form (indicating which one of the two), and then giving the reduced form, labeling it as such. For example, if you need to find the reduced row echelon form of the matrix A, for whatever reason, you may write

$$\mathrm{rref}(A) = \cdots$$

where $\cdots$ is the reduced row echelon form of the matrix. When calculating the determinant of a matrix, you should indicate the cofactor expansion you are using to reduce the problem to finding determinants of 2×2 matrices.