21-241 Lec5 Exam 2 Study Guide

If you understand and can do the following things, you should do well on the second exam. Remember, it is closed book, closed notes, no electronic devices except internet-less calculators, etc. You should also be comfortable writing mathematical proofs. On the exam, your proofs should meet the following specifications:

1. everything stated should be true and explained where appropriate (Yes, I know this is vague, but sometimes this is a stylistic decision. When in doubt, explain why your statement is true.),
2. the proof needs to prove the actual statement in question, and
3. the proof needs to be written in clear mathematical English (that is, there should be words explaining what you are doing).

1 Matrices

You should know/be able to do

- find the inverse of a matrix, if it exists,
- use the properties of inverses of matrices from section 3.3,
- elementary matrices, and how they are related to row operations,
- use the LU factorization of a matrix to solve $LU\mathbf{x} = \mathbf{b}$,
- the Fundamental Theorem of Invertible Matrices (see page 296).

2 Subspaces

You need to be comfortable with/comfortable with using

- the definition of a subspace of $\mathbb{R}^n$ and showing a subset S of $\mathbb{R}^n$ is a subspace,
- row spaces and column spaces of matrices,
- null space of a matrix,

- the definition of a basis, and showing a set of vectors is a basis,
- the dimension of a subspace,
- the Rank Theorem.

3 Linear Transformations

You need to be comfortable with/comfortable with using

- the definition of a linear transformation,
- showing a given function $T : \mathbb{R}^n \mapsto \mathbb{R}^m$ is a linear transformation,
- the relationship between linear transformations between $\mathbb{R}^n$ and $\mathbb{R}^m$ and $m \times n$ matrices
- how to find the standard matrix of a linear transformation,
- composition and inverses of linear transformations.

4 Eigenvalues and Eigenvectors

You need to know/be able to handle

- the definition of eigenvalues and eigenvectors for an $n \times n$ matrix,
- finding determinants of matrices,
- the properties of determinants,
- the characteristic polynomial of a matrix and using it to find the eigenvalues,
- finding bases for eigenspaces,
- given eigenvalues and eigenvectors of a matrix and $\mathbf{x}$ which is a linear combination of the given eigenvectors, finding $A^k\mathbf{x}$.

5 Note About Using Calculators

As I have said in class, you are allowed to use graphing calculators, so long as they do not have the capability of connecting to any computer network. As such, it is not expected on the exam that you do row reductions by hand or calculate inverses of matrices by hand (granted, this second one is a case of row reduction). When you are showing your work on the exam, it is sufficient to make clear which matrix you are putting into row echelon form or reduced row echelon form (indicating which one of the two), and then giving the reduced form, labeling it as such. For example, if you need to find the reduced row echelon form of the matrix A, for whatever reason, you may write

$$\mathrm{rref}(A) = \cdots$$

where $\cdots$ is the reduced row echelon form of the matrix. When calculating the determinant of a matrix, you should indicate the cofactor expansion you are using to reduce the problem to finding determinants of 2×2 matrices.