Homework 7–21-124, Calculus II for Biologists and Chemists

Name: ______________________________
Section: ______________________________

Instructions: Complete the following problems, clearly labeling the problems. Staple this sheet, with your name and section filled in, to the top of your work. Failure to attach this sheet will result in a one point deduction in the grade. The assignment will be graded out of twenty points.

DUE: Friday, March 25, 2016

Book Problems

- Section 11.1: 4, 20, 22, 24, 28, 30, 32, 36, 42, 68(a and b only)
- Section 11.2: 4, 8, 12, 20, 22, 24, 28
- Section 11.3: 2, 8, 12

Other Problems

For these problems, we will consider the *inhomogenous* linear system

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + \mathbf{f}(t) \tag{1}$$

where $\mathbf{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, A is a 2×2 matrix with constant entries, and $\mathbf{f}(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$ for some real-valued functions f_1 and f_2.

1. Let $\mathbf{x}_i$ be a solution to (1) and suppose $\mathbf{x}_h$ solves the homogeneous problem

   $$\frac{d\mathbf{x}}{dt} = A\mathbf{x}. \tag{2}$$

   Show that $\mathbf{x} = \mathbf{x}_h + \mathbf{x}_i$ solves (1).

2. The general solution to (1) is given by $\mathbf{x}_g = \mathbf{x}_i + \mathbf{x}_{hg}$ where $\mathbf{x}_i$ is a particular solution to (1) and $\mathbf{x}_{hg}$ is the general solution to (2). Consider the problem

$$(3) \qquad \begin{cases} \frac{dx_1}{dt} = x_1 + x_2 - 5t + 2 \\ \frac{dx_2}{dt} = 4x_1 - 2x_2 - 8t - 8. \end{cases}$$

Note that $\mathbf{x}_i = \begin{bmatrix} 3t+2 \\ 2t-1 \end{bmatrix}$ solves (3). Find the general solution to (3).