21-124 **Groupwork 9** **March 24, 2016**

Answer the questions below. You may answer in the space provided. You may use the back or a separate sheet of paper if you need more space. You are to work in groups of no more than four people. Make sure to enter the names of your groupmates below.

Name: _____

Section: _____

Group Members: _____

1. The predator-prey model discussed in class has a key short-coming: it assumes that the environment can support arbitrarily many members of the species of prey. For a certain environment and predator and prey species, the population of prey, $x_1(t)$ and population of predator, $x_2(t)$ is modeled by

$$\begin{cases} \frac{dx_1}{dt} = 2x_1\left(1 - \frac{x_1}{10}\right) - 3x_1x_2 \\ \frac{dx_2}{dt} = x_1x_2 - 3x_2 \end{cases} \tag{1}$$

   (a) (2 points) What differential equation does the population of prey obey in the absence of predators? What limit does this put on the population of prey?

   (b) (2 points) Find the equilibrium solution(s) of (1).

   (c) (2 points) Find the Jacobian for the system in (1)

   (d) (2 points) Use the Jacobian to analyze any equilibria for the system.

2. (2 points) For the system of differential equations

$$\begin{cases} \frac{dx_1}{dt} = 2x_1 - 3x_2 \\ \frac{dx_2}{dt} = 6x_2 - 4x_2, \end{cases}$$

   find a conserved quantity and verify it is a conserved quantity.