

21-370 DISCRETE TIME FINANCE
WRITTEN HOMEWORK — WEEK #14

2016 FALL

You may wish to review problem 2.10 from the Week #6 Homework assignment before beginning these problems.

- (1) Consider a 4-period binomial asset pricing model with parameters $u = 2$, $d = \frac{1}{2}$, $r = \frac{1}{4}$, and $S_0 = 4$. The stock in this model pays a dividend at time 3 that is $1/4$ the share price at that time:

$$d_3(\omega) = \frac{1}{4}S_3(\omega).$$

When the dividend is paid, the share price drops by the amount of the dividend. That is

$$S_{3+}(\omega) = S_{3-}(\omega) - d_3(\omega) = u^{\#H(\omega)}d^{\#T(\omega)}S_0 - d_3(\omega).$$

No other dividends are paid. At time 4, the share price of the stock is $uS_{3+}(\omega)$ or $dS_{3+}(\omega)$, depending on the outcome of the fourth toss.

- (a) Find the arbitrage free price at time 0 of an American call option on one share of this stock with maturity 4 and strike price 10. Is there a situation in which the option is exercised before maturity?
- (b) What is the delta hedging formula at time 3? Does it need to be modified (from eq. 1.1.9) because of the dividend? Explain why or why not.
- (c) What is the delta hedging formula time 2? Does it need to be modified because of the dividend? Again, explain why or why not.
- (2) Consider again the 4-period binomial asset pricing model with parameters $u = 2$, $d = \frac{1}{2}$, $r = \frac{1}{4}$, and $S_0 = 4$. In this case stock in this model pays a dividend at time 3 that is $1/4$ the share price at *time 2*:

$$d_3(\omega) = \frac{1}{4}S_2(\omega).$$

As before, when the dividend is paid, the share price drops by the amount of the dividend. That is

$$S_{3+}(\omega) = S_{3-}(\omega) - d_3(\omega) = u^{\#H(\omega)} d^{\#T(\omega)} S_0 - d_3(\omega).$$

No other dividends are paid. At time 4, the share price of the stock is $uS_{3+}(\omega)$ or $dS_{3+}(\omega)$, depending on the outcome of the fourth toss.

- (a) Find the arbitrage free price at time 0 of an American call option on one share of this stock with maturity 4 and strike price 10. Compared to the computation in the previous problem, this computation in this problem has an additional complication. What is it?
- (b) What is the delta hedging formula at time 3? Does it need to be modified (from eq. 1.1.9) because of the dividend? Explain why or why not.
- (c) What is the delta hedging formula time 2? Does it need to be modified because of the dividend? Again, explain why or why not.