Final Exam Review

Closed book and notes; only the following calculators will be permitted: TI-30X IIS, TI-30X IIB, TI-30Xa.

1. Consider a market in which three currencies (dollars, pounds and yen) are traded. Suppose that currently ($t = 0$) the value of one pound in dollars is $E_d^p = 1.5$ and the value of one dollar in yen is $E_y^d = 95$. The interest rates for pounds and dollars between $t = 0$ and $t = 1$ are $r_d = .03$ and $r_p = .05$ respectively. The forward exchange rate between pounds and yen, for trades at $t = 1$ is $F_y^p = 120$.
   (a) Find the value E_y^p of one pound in yen.
   (b) Find the forward exchange rates F_p^d and F_y^d
   (c) A company operating in the US must make three payments of £10,000 to a British contractor at the times $t = 1, 2, 3$. The company asks you to make these three payments in exchange for three payments of A dollars at the times $t = 1, 2, 3$. You may assume that the interest rates for both dollars and pounds are constant. (e.g. any investment of P dollars for a one year period grows to $(1 + r_d)P$.) What is the arbitrage free value of A?

2. Consider the following fixed income securities:
   - B, a coupon bond with face value \$1000, coupon rate $q[1] = .05$ and maturity $T = 2$.
   - A^1, and annuity making two payments per year of \$500, and maturity $T = 2$.
   - A^2, an annuity making payments at times $t = \frac{1}{2}$ and $t = \frac{3}{2}$, each of \$1000.

   (a) Given the price of the bond is $\mathcal{P}(B) = \$1043.57$ and $R_*(2) = .0275$, find $R_*(1)$.
   (b) If, in addition to the conditions in part (a), the price of the second annuity is $\mathcal{P}(A^2) = \$1958.67$, find $\mathcal{P}(A^1)$.
   (c) Assuming all the information from parts (a) and (b), what is the internal rate of return for B.

3. Consider a financial model with a single stock, a bank and two trading times, $t = 0$ and $t = 1$. The stock has initial price $S_0 = \$100$ per share, and at $t = 1$, the share price is known to be either \$120 or \$90. The bank offers a one period interest rate of $r = .05$ for either borrowing or investing.

(a) An investor holds a portfolio X consisting of one share of the stock, one put with strike price $K_P = 111$ and a short position on a call with strike price $K_C = 97$. What are the two possible values X_1 of the portfolio at time $t = 1$.

(b) What is the initial value P_0 of the put with strike price \$111?

(c) What is the initial value X_0 of the investor's portfolio?

4. Consider a financial model with one stock, a bank and two trading times $t = 0$ and $t = 1$. The stock has initial price $S_0 = \$32$ per share, and at $t = 1$, the share price is known to be one of three values: \$50, \$30 or \$10. The bank offers a one period interest rate of $r = .25$ for either borrowing or investing.

   (a) Show that this model is arbitrage-free. Is it complete?

   (b) Find all the possible arbitrage free prices of a put with strike price $K = \$40$.

   (c) Suppose that the put from part (b) is added to the model with an initial value of $P_0 = 5$. Is the resulting model arbitrage-free? Is it complete?

5. Consider a finite one period financial model with sample space $\Omega = \{\omega_1, \ldots, \omega_n\}$, stocks $S_1, \ldots, S_k$, and one-period interest rate r. Let $\overline{\mathbb{P}}$ be a probability measure on Ω. [i.e. $\overline{\mathbb{P}}(\omega) > 0$ for $\omega \in \Omega$ and $\sum_{\omega\in\Omega} \overline{\mathbb{P}}(\omega) = 1$.]

   Show that $(1 + r)X_0 = \mathbb{E}^{\overline{\mathbb{P}}}(X_1)$ for every portfolio X if and only if $(1 + r)S_0^j = \mathbb{E}^{\overline{\mathbb{P}}}(S_1^j)$ for each $j = 1, \ldots, k$.

6. Consider a financial model with one stock, a bank and three trading times $t = 0$, $t = 1$ and $t = 2$. The bank offers an interest rate of $r = \frac{1}{5}$ for borrowing or investment over either of the two periods (i.e. from 0 to 1 or from 1 to 2). The stock has initial price $S_0 = \$25$ per share. The stock pays no dividends. At $t = 2$ the stock will have one of two values, \$54 or \$18.

   (a) Consider a derivative security V that, at $t = 0$ allows you to choose between one share of the stock, or a zero coupon bond with face value \$36 and maturity $T = 2$. Show that the value of this security at $t = 1$ is $V_1 = 30 + \max\{0, C_1 - P_1\}$, where C and P represent a call and a put on the stock with strike price $K = \$36$ and expiration date $T = 2$.

   (b) Suppose that it is known that a call on the stock with strike price \$45 and expiration date $T = 2$ will have one of two values at $t = 1$, \$5 or \$2.50. In this case, what are the possible values of V_1, the price of the security at $t = 1$?

   (c) What is the value V_0 of the security at $t = 0$?

7. Consider a one-period binomial model with $u = 3$, $d = \frac{1}{3}$, $r = \frac{1}{6}$, $S_0 = 30$, $\mathbb{P}(H) = \frac{3}{4}$ and $\mathbb{P}(T) = \frac{1}{4}$. An investor with utility function $U(x) = \sqrt{x}$ has initial capital \$1000. Find the terminal capitals $\{X_1(H), X_1(T)\}$ that maximize the expected utility of his portfolio. How should the investor allocate their capital to maximize it's expected utility?

8. Consider a one-period binomial model with a stock:

$$S_0 = 100, \quad S_1(H) = 140, \quad S_1(T) = 90$$

and interest rate $r = .2$. The (real-world) probabilities of each outcome are $\mathbb{P}(H) = \frac{2}{3}$ and $\mathbb{P}(T) = \frac{1}{3}$

(a) An investor with utility function $U(x) = \ln(x)$ has \$10,000 to invest. How should he allocate his capital between the stock and the bank account in order to maximize his expected utility?

(b) Answer the same question for an investor with utility function $U(x) = -\frac{1}{x}$.