Exam #2 Review

1. Two different annuities A1 and A2 and a coupon bond B are being issued today. Their characteristics are given below.

   - A1 has maturity 2 years and will make payments of $1,000 twice per year. The current price of this annuity is $3818.69.
   - A2 has maturity 2 years and will make payments of $200 once per year. This annuity has an internal rate of return $R_I = 3.75\%$
   - B has maturity 2 years, face value $1,000 and will make coupon payments twice per year at the nominal coupon rate q[2] = 4%. The current price of this bond is $1005.39.

   (a) What is the effective interest rate $R_*(2)$?

   (b) If $R_*(.5) = 3.5\%$, what is the effective interest rate $R_*(1.5)$?

2. An annuity paying $1000 once per year for 10 years has an effective internal rate of return $R_I = 5\%$. A zero coupon bond with face value $10,000 and maturity 10 years is is available for $Z_0 = \$5584$.

   Two companies wish to enter into an interest rate swap, where payments will be made once per year for 10 years. If the fixed payment is to be $10,000 each year, what must be the notional principal amount F of the swap.

3. A coupon bond making coupon payments once per year has face value $1000, coupon rate $q[1] = 2.5\%$, and maturity $T = 10$ years. This bond is being sold today for $751.44. Find the forward price for purchase of this bond at $\tau = 2$, just after the second coupon payment. Assume the existence of an ideal bank with constant effective interest rate $R = 6\%$.

4. The spot price for one ounce of gold today ($t = 0$) is $G_0 = \$1500$ and the forward price to purchase one ounce of gold in six months is $\mathcal{F} = \$1584$. The discount factor for six months is $D(.5) = .95$. If you are able to store gold safely for six months at a cost of $2 per ounce, payable at $t = 0$, is there an opportunity for you to make a profit?

5. An certain astute trader has the opportunity to buy or sell a limited number of put options on a stock at the price $\hat{P}_0 = \$5$. The strike price of the option is K = $48 and its exercise date is T = 1 year. The current price of the stock is $S_0 = \$50$ and the stock will pay a dividend $d = \$1$ per share in six months. (No other dividend payments will be made in the time interval [0, 1].) Assume that the discount factors for six months and one year are given by D(0.5) = 0.98 and D(1) = 0.95, respectively. Will the trader buy the options, sell the options, or decide that there is no advantage in trading this security? If he does trade the options, what will his strategy be? What will be his profit (per option) from this transaction?

6. A certain stock is selling today (t=0) for S_0 dollars per share. The forward price for delivery of one share of this stock at time T is $\mathcal{F}$ dollars. It is known that time τ $(0 < \tau < T)$ the stock will pay a dividend of d dollars. Additionally, at time σ $(0 < \sigma < T)$ the stock will pay a dividend equal to αS_σ (where S_σ is the price for one share at time σ).

   Find an expression for $\mathcal{F}$ in terms of S_0, T, τ, d, σ, α, and whatever discount factors are necessary. Does it matter whether $\tau < \sigma$ or $\sigma < \tau$? If so, find an expression for each case.

7. A fixed income security will make a payment of P dollars six months from now, and another payment of $2P$ dollars one year from now. This security makes no other payments. The security may be purchased today for $\hat{\mathcal{P}} = \frac{5}{2}P$. What is the internal rate of return for this security? Does it depend on the value of P?

8. Heating oil is available for purchase today at a cost of \$2.53 per gallon. The forward price for delivery of oil one year from today is \$2.94 per gallon.

   You have a friend who knows a guy who will allow you to safely store up to 10,000 gallons of oil for one year at a cost of \$0.02 per gallon per month, but you must pay all of the storage costs at the start of the year.

   The discount factor for borrowing or investing for one year is $D(1) = .97$.

   (a) Are you able to realize an arbitrage profit in this situation? If not, why not? If so, what is the amount of the profit you can achieve at time $t = 1$?

   (b) What can you say about market rates for storing oil?

9. A coupon bond with face value \$1,000 which makes 2 coupon payments per year for 10 years at a coupon rate of $q[2] = 4\%$ is available at the arbitrage-free price of $\mathcal{P}_0^B = \$926.60$.

   An annuity which makes payments of \$200 twice per year for 5 years is available at the arbitrage free price of $\mathcal{P}_0^A = \$1753.17$. The annuity payments are made at the same time as the first ten coupon payments for the bond.

   The forward price to purchase the bond at $t = 5$, just after the coupon payment at that time, is $\mathcal{F} = \$958.85$.

   What is the discount factor $D(5)$?

10. A certain stock will pay will pay a dividend at time $\tau > 0$. The amount of the dividend will be a fraction α of the share price at time τ, i.e. $d_\tau = \alpha S_\tau$.

    The stock will pay will pay a second dividend at time $\sigma > \tau > 0$. The amount of the dividend will be a fraction β of the share price at time σ, i.e. $d_\sigma = \beta S_\sigma$.

    Based on our in-class discussions of forward prices for dividend paying stocks, one might guess that the forward price for purchase of 1 share of this stock at time $T > \sigma > \tau > 0$ ought to be

    $$\mathcal{F} = \frac{(1-\alpha)(1-\beta)S_0}{D(T)}.$$

    Is this correct? Explain why it is or is not. A complete explanation probably requires a discussion of a replicating portfolio.