21-270 **Introduction to Mathematical Finance**

Homework #9: Due Friday, March 20 **Spring 2020**
Correction: Due Monday, March 23

1. Assume that there is an ideal money market with effective annual rate $R = 5\%$.

   A stock S is trading today at the initial price $S_0 = \$67$ per share. The stock will pay a dividend two months from today equal to $\alpha = 2\%$ of the value of one share of stock on that day (i.e. 2% of the value just before the dividend is paid — After the dividend is paid, we assume the share price will fall by the amount of the dividend).

   A European call option C on the stock with exercise date six months from today and strike price $K = \$75$ is trading today at the initial price $C_0 = \$9$.

   Assuming that there is no arbitrage, find the initial price P_0 of a European put option P on the stock with exercise date and strike price equal to those of the call option described above.

2. Assume that there is an ideal money market with effective annual rate $R = 4\%$.

   A stock S is trading today at the initial price $S_0 = \$85$ per share. The conpany has announced that the stock will pay a dividend two months from today, but the amount of the dividend, δ, has not yet been announced.

   A European call option C on the stock with exercise date eight months from today and strike price $K = \$75$ is trading today at the initial price $C_0 = \$13.42$.

   A European put option P on the stock with the same exercise date and strike price as the call option is trading today at the initial price $P_0 = \$4.46$.

   Assuming there is no arbitrage, determine the value of δ that is predicted by the market.

3. [**Exercise 3.11**]

   (a) Prove Remark 3.28.

   (b) How should equations (3.10) and (3.11) in the posted notes be modified if it is known that the stock pays a dividend of δ dollars per share at time $\tau \in (0, T)$? Assume that δ and τ are known at time 0 and that there will be no other dividend payments made between time 0 and time T.

   (c) How should equations (3.10) and (3.11) in the posted notes be modified if the stock pays a dividend of amount $\delta = \alpha S_{\tau-}$ per share at time $\tau \in (0, T)$? Assume that α is known at time 0 and that there will be no other dividend payments made between time 0 and time T. Also assume that at the time the dividend is paid, the stock price will immediately drop by the amount of the dividend, i.e. $S_{\tau-} = \delta + S_{\tau+}$.

4. [**Exercise 3.13**] Consider the chooser option of Example 3.29 with $S_0 = \$100$, $\tau = 6$ months, $T = 1$ year, $K = \$105$¡ and $R = .05$. Let P and C denote European put and call options on S with expiration date $T - 1$ year and strike price $K = \$105$. Let $\hat{P}$ denote a

European put on S with expiration date 6 months and strike price $\$105(1.05)^{1/2}$. Assume that

$$C_0 = \$8.02, \quad \hat{P} = \$5.61,$$

and that the stock does not pay dividends, so that $V_0 = \$13.63$. At $t = 0$ a client purchases 10,000 of the chooser options described above from a broker for $10.000V_0$ plus a commission. In order to hedge her short position, the broker purchases 10.000 each of C and $\hat{P}$ and holds them for 6 months. At time $t = 6$ months the prices of S and C are $S_{.5} = \$110$ and $C_{.5} = \$10.49$. Since $S_{.5} > \$103(1.05)^{-1/2}$, we know that $C_{.5} > P_{.5}$, so that the client should decide to choose the call for all of the shares of V. However, the client has insider information and knows that, although it has not been announced yet, a regulatory agency has made a decision that will have a significant and negative impact on the profits of the company that issued the stock. Once the decision is made public, the stock price will almost certainly make a dramatic drop. The client is convinced that the stock price will not recover in six months and decides to choose the puts for all 10,000 options, even though the calls are currently moe valuable. In order to keep her short position hedged, the broker sells all 10,000 calls C and purchases 10,000 puts P.

(a) The broker will have cash left over at $t = 6$ months that can safely be consumed (or invested). How much extra cash will the broker have at $t = 6$ months?

(b) Suppose that $S_1 = \$69.78$. What will be the value of the client's portfolio at time 1?

(c) Assuming that $S_1 = \$69.78$, both the broker and the client have made a lot of money on this deal. Where did this money come from? In particular, do you think that there is a victim here? (There are currently strict regulations in teh United State3s governing securities trades based on insider information. It is worth noting that experts have differing views on insider trading.)