21-270 **Introduction to Mathematical Finance** **D. Handron**

Week #2 Written Assignment: Due on Wednesday, January 22. **Spring 2020**

1. Let $T > 0$ be given. Let C denote a call option (on a stock S) with exercise date T and strike price $K_c = \$75$. Let P denote a European put option on the same stock with exercise date T and strike price $K_p = \$67$. Let S_T denote the price per share of the stock at time T and let C_T and P_T denote the value per share of the call and put options (respectively) at time T.

   Find C_T and P_T

   (a) if $S_T = \$58.75$.

   (b) if $S_T = \$69.37$.

   (c) if $S_T = \$78.21$.

2. Shares of stock of the QRS Company may be bought or sold today for $S_0^1 = \$20$. A put option on this stock with expiration date $T = 1$ and strike price $K^1 = \$12$ may be bought or sold for $P_0^1 = \$6$.

   Shares of stock of the XYZ company may be bought or sold today for $S_0^2 = \$10$. A call option on this stock with expiration date $T = 1$ and strike price $K^2 = \$9$ may be bought or sold for $C_0^2 = \$4$.

   An investor decides to buy 10 shares of QRS, 5 of the put options on QRS, 20 shares of XYZ and 10 of the call options on XYZ.

   (a) What is the value of the investor's portfolio at time $t = 0$?

   (b) What is the value P_1^1 of one of the put options at time $t = 1$ if $S_1^1 = \$27$? If $S_1^1 = \$7$?

   (c) What is the value C_1^2 of one of the call options at time $t = 1$ if $S_1^2 = \$14$? If $S_1^2 = \$4$?

   (d) What is the value of the investor's portfolio at time $t = 1$ if $S_1^1 = \$27$ and $S_1^2 = \$14$? If $S_1^1 = \$7$ and $S_1^2 = \$4$?

3. Stock of the ABC company is currently trading at the price of $S_0 = \$40.00$ per share. European call options on the stock with $T = 1$ years and $K = \$45.00$ are trading at the current price of $C_0 = \$0.40$ per option. European put options on the stock with $T = 1$ years and $K = \$35.00$ are trading at the current price of $P_0 = \$0.34$ per option. At $t = 0$ an investor purchases 100 of the call options described above and 200 of the put options. He also takes a long position on a forward contract to purchase of 50 shares of stock at the delivery date $T = 1$ and the forward price of $\mathcal{F} = \$41.00$ per share. Assume that the investor does not make any trades between $t = 0$ and $t = 1$.

   (a) What is the initial value of this investors portfolio?

   (b) Sketch a graph of the value of this investors portfolio as a function of $S_!$, the stock price at time $t = 1$.

(c) For what values of S_1 does the investor make a profit? For what values of S_1 does the investor incur a loss?

4. Stock of the PDQ company is selling today at the price $S_0 = \$60$ per share. European put options on the stock with exercise date $T = 1$ and strike price $K = \$60$ are selling today at the price $P_0 = \$4.50$ per option. An investor with no initial capital believes that the stock price is going to drop, so she constructs a portfolio by selling short 300 shares of stock and using all of the proceeds of the short sale to purchase options of the type described above. She makes no transactions between $t = 0$ and $t = 1$. Find the value of her portfolio at $t = 1$ (i.e. find X_1) if

   (a) $S_1 = \$30$.

   (b) $S_1 = \$90$.