21-270 **Introduction to Mathematical Finance**

Homework #11: Due Wednesday, April 8 **Spring 2020**

1. **[Exercise 4.4]** Let $(\Omega, \mathbb{P})$ be a finite probability space and let $X : \Omega \to \mathbb{R}$ be a random variable. Assume that $\mathbb{P}(\omega) > 0$ for all $\omega \in \Omega$. Show that if $\mathrm{Var}(X) = 0$ then X is constant, i.e. there. exists $c \in \mathbb{R}$ such that $X(\omega) = c$ for all $\omega \in \Omega$.

2. **[Exercise 4.5]**

3. **[Exercise 4.6]**

4. **[Exercise 4.9]** Let $(\Omega, \mathbb{P})$ be a finite probability space and let $X, Y : \Omega \to \mathbb{R}$ be random variables. Show that
$$|\mathrm{Cov}(X, Y)| \leq \mathrm{Sd}(X)\mathrm{Sd}(Y).$$
Suggestion: Assume first that $\mathrm{Var}(X) > 0$. Let $\lambda \in \mathbb{R}$ be an arbitrary constant. Expand out
$$\mathrm{Var}(Y - \lambda X)$$
and notice that this quantity must be greater than or equal to zero. Then set
$$\lambda = \frac{\mathrm{Cov}(X, Y)}{\mathrm{Var}(X)}.$$
To complete the proof, show that if $\mathrm{Var}(X) = 0$, then $\mathrm{Cov}(X, Y) = 0$.

5. **[Exercise 5.4]**

6. **[Exercise 5.7]**