Week #6 Written Assignment: Due on Frinday, October 4.

1. **(Boundary Value Problems I)** Boundary value problems can specify values for the *derivative* of the solution at either (or both) endpoints, instead of values of the function itself. Solve the following boundary value problems:

   (a) $y'' + y' - 12y = 0$, with boundary conditions $y(-1) = 1$, $y'(1) = 3$.

   (b) $y'' - 6y' + 9y = 0$, with boundary conditions $y'(0) = 0$, $y(3) = 0$.

   (c) $y'' + 4y' + 4y = 0$, with boundary conditions $y'(0) = 1$, $y'(2) = -1$

2. **(Boundary Value Problems II)** Initial value problems are fairly straightforward. We have a theorem that tells us that for "nice" differential equations, there will be a unique solution to any initial value problem.

   Boundary value problems may or may not have a solution, even for "nice" equations. When there is a solution, it may have a single solution, or infinitely many.

   (a) Show that a boundary value problem of the form

   $$y'' + ky = 0, \quad y(0) = 0, y(L) = 0$$

   with $k \in \mathbb{R}$ and $L > 0$ always has the *trivial solution*, $y(x) = 0$.

   (b) In this part of the problem, we take $L = 1$ and ask:

   For what values of k will the boundary value problem

   $$y'' + ky = 0, \quad y(0) = 0, y(1) = 0$$

   have a non-trivial *solution (i.e. a solution that is not the constant $y(x) = 0$)?*

3. **(Method of undetermined coefficients)**

   (a) Show that if $y_1(x)$ is a solution to

   $$ay'' + by' + cy = g_1(x)$$

   and $y_2(x)$ is a solution to

   $$ay'' + by' + cy = g_2(x),$$

   then the sum $y_1(x) + y_2(x)$ is a solution to

   $$ay'' + by' + cy = g_1(x) + g_2(x).$$

   (b) Using the result from part (a) find a particular solution for the equation

   $$y'' + 4y' + 5y = x^3 + \cos(3x).$$