DIFFERENTIAL EQUATIONS
WRITTEN HOMEWORK WEEK #4

2012 SPRING

(1) In class we examined the equation

$$\frac{dy}{dt} = t + y^2$$

using Euler's method. In this problem, you will continue that investigation, but with a different initial condition. Let ϕ denote the solution to the initial value problem

$$\frac{dy}{dt} = t + y^2, \quad y(0) = 1.$$

(a) Use Euler's method with step size $\Delta t = 1$ and $(t_0, y_0) = (0, 1)$ to find the value $y_1 \simeq \phi(1)$. Do the computations "by hand." You can use a calculator for arithmetic operations, but work through each step yourself.

(b) Use Euler's method with step size $\Delta t = .5$ and $(t_0, y_0) = (0, 1)$ to find the value $y_2 \simeq \phi(1)$. Do the computations "by hand." You can use a calculator for arithmetic operations, but work through each step yourself.

(c) Use Euler's method with step size $\Delta t = .25$ and $(t_0, y_0) = (0, 1)$ to find the value $y_4 \simeq \phi(1)$. Do the computations "by hand." You can use a calculator for arithmetic operations, but work through each step yourself.

(d) Use Euler's method with step size $\Delta t = .1$ and $(t_0, y_0) = (0, 1)$ to find the value $y_{10} \simeq \phi(1)$. You may wish to automate this process using technology — Maple, Mathematica or Dfield, for example. You can use Dfield and read an approximate value from the graph, if you like. Be sure to indicate how you arrived at your value.

The approximate values for $\phi(1)$ seem to be increasing, rather than approaching a limit as the Δt is reduced.

(e) Note that $y^2 \leq t + y^2 \leq 1 + y^2$ when $0 \leq t \leq 1$. Let ϕ_0 be a solution to $\frac{dy}{dt} = y^2$ satisfying $\phi_0(0) = 1$, and ϕ_1 be a solution to $\frac{dy}{dt} = 1 + y^2$ satisfying $\phi_0(0) = 1$. Explain why it is plausible to think that $\phi_0(t) \le \phi(t) \le \phi_1(t)$ when $0 \le t \le 1$.

(f) Use the technique for separable differential equations to find explicit expressions for $\phi_0(t)$ and $\phi_1(t)$. What is the domain of definition for these solutions?

(g) What can you now say about the domain of definition of $\phi(t)$? How can you now interpret the behavior of the numerical approximations in parts (a)-(d)?

(2) Problem **2.5.20**. Be sure to read the paragraph "Harvesting a Renewable Resource" immediately preceeding this problem.

(3) Problem **2.5.25**. Be sure to read the paragraph "Bifurcation Points" immediately preceeding this problem.

(4) One additional problem to be added...