DIFFERENTIAL EQUATIONS
WRITTEN HOMEWORK — WEEK #2

2010 FALL

(1) Draw a direction field for the given differential equations. Based on the direction field, determine the behavior of the solution as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe the dependency.
   (a) **1.1.2.** $y' = 2y - 3$ (Draw the direction field by hand)
   (b) **1.1.30.** $y' = 3\sin(t) + 1 + y$ (Use `(dfield)` to draw direction field)

(2) **2.2.23.** Solve the initial value problem

$$\frac{dy}{dt} = 2y^2 + ty^2, \quad y(0) = 1,$$

and determine where the solution attains its minimum value

Additionally, determine the interval on which the solution is valid.

(3) This problem builds on an example that was discussed in class. We have looked at the direction field for the differential equation

$$\frac{dy}{dt} = t + 2y.$$

   (a) Show that the *isoclines* for this equation are of the form

$$y = -\frac{1}{2}t + \frac{m}{2},$$

   where m indicates the slope of the direction field lines along the isocline. [Note: We went over this in class, so this should not be difficult.]
   (b) The isoclines for this equation can only be crossed in one direction, i.e. if one solution passes from below the isocline to above, then any solution that crosses that isocline must pass from below to above. For what values of m will the solutions cross from below the isocline to above? For what values of m will the solutions cross from above the isocline to below?

(c) For what value of $m = m_0$ will solutions be unable to cross the corresponding isocline in either direction? What is the behavior of solutions with initial conditions on this isocline?

(d) Show that the isocline corresponding to $m = m_0$ is the graph of a solution to the differential equation. [Note: This is best done by finding a formula for this solution?]