Week #12 Written Assignment: Due on *Friday*, November 15.

1. Consider the 3-dimensional system of linear equations

$$X' = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} X$$

   (a) Find a fundamental set of solutions for this system. Note that -1 is one of the eigenvalues.

   (b) Find the general solution, and use it to find the solution satisfying

$$X(0) = \begin{bmatrix} -4 \\ 3 \\ 2 \end{bmatrix}$$

2. Consider the 3-dimensional system of linear equations

$$X' = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{bmatrix} X$$

   (a) Find a fundamental set of solutions for this system. Note that $+1$ is one of the eigenvalues.

   (b) Find the general solution, and use it to find the solution satisfying

$$X(0) = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

3. (Taken from Boyce & DiPrima) Consider the 3-dimensional system of linear equations

$$X' = AX = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{bmatrix} X$$

   (a) Show that the three eigenvalues of the coefficient matrix, A, are $\lambda_1 = \lambda_2 = \lambda_3 = 2$. This is an eigenvalue of multiplicity 3.

   (b) Show that all the eigenvectors of the coefficient (corresponding to the eigenvalue 2) are multiples of

$$V = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

   (c) Using the information from parts (a) and (b), find one solution to the system.

(d) Find a generalized eigenvector, W_1, of the coefficient matrix, A, by solving

$$(A - 2I)W_1 = V$$

Use this generalized eigenvector and the other information you have found to write down a second solution to the system.

(e) Show that if V and W are as above, and W_2 is a vector satisfying

$$(A - 2I)W_2 = W_1$$

then

$$X = \frac{t^2}{2}e^{2t}V + te^{2t}W_1 + e^{2t}W_2$$

is a solution to the system.

(f) Find a vector W_2 satisfying $(A - 2I)W_2 = W_1$, and use it to write a third solution to the system.

(g) Compute the Wronskian for the three solutions you found in parts (c), (d), and (f). Is it non-zero? Do these three solutions form a fundamental set?