21-260 **Differential Equations** **D. Handron**

Week #1 Written Assignment: Due on Friday, May 22.

1. Given the differential equation and function

$$\frac{dy}{dt} + 20y = 24, \quad \phi(t) = \frac{6}{5} - \frac{6}{5}e^{-20t}$$

   do the following:

   (a) Identify the dependent and independent viariable(s).

   (b) Classify the equation as linear/non-linear, and ODE/PDE. What is the order of the equation?

   (c) Show that $y = \phi(t)$ is a solution to the differential equation.

2. Solve the initial value problem

$$y' = 2y^2 + xy^2, \quad y(0) = 1$$

   and determine where the solution attains its minimum value.

3. Using the direction field below for the differential equation

$$\frac{dy}{dx} = x^2 - y^2$$

   sketch solution curves satisfying the following initial conditions:

$$y(0) = 0, \quad y(-1) = \frac{1}{2}, \quad y(0) = -1, \quad y(1) = 0$$

4. Consider the differential equation

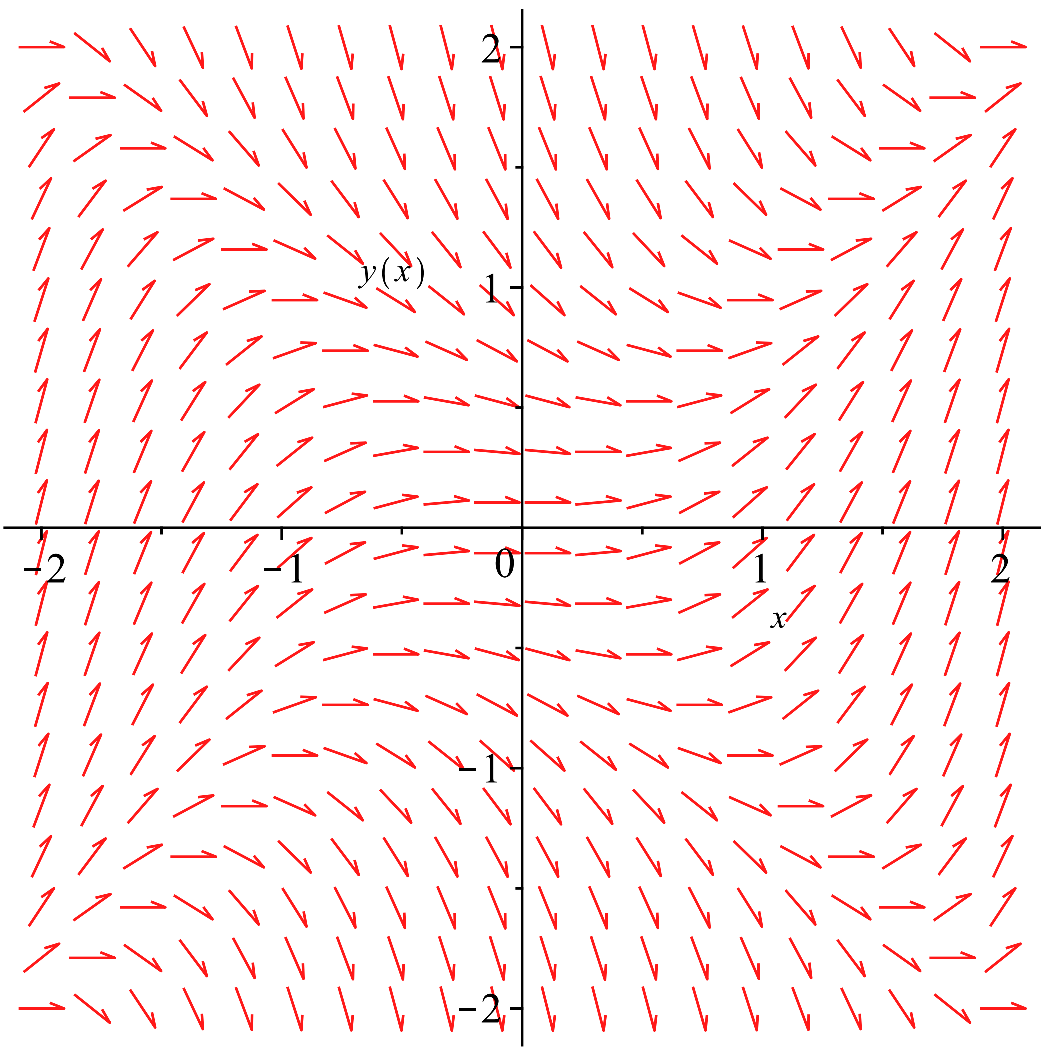
$$\frac{dy}{dt} = 1 - ty.$$

   (a) Find the isocline corresponding to slopes $m = 0, +1, -1$

   (b) Use the isoclines you found in part (a) to sketch a direction field for the equation. Include as many additional isoclines as you need to get a good sense for the behavior of solutions.

   (c) Using the direction field you created in part (b) sketch some solution curves for this differential equation.

5. (Note: Don't make this problem harder than it really is, which is not too hard. It is a foreshadowing of things to come.)


y(x)
2
1
−1
−2
−2
−1
0
1
2
x

(a) Show that the solution

$$y(t) = \frac{1}{\mu(t)} \left[C + \int_{t_0}^{t} \mu(s) g(s)\, ds \right]$$

of the differential equation

$$y' + p(t)y = g(t)$$

can be written in the form $y(t) = Cy_1(t) + y_2(t)$. Identify the functions $y_1(t)$ and $y_2(t)$.

(b) Show that $y_1(t)$ is a solution to the homogeneous differential equation

$$y' + p(t)y = 0.$$

(c) Show that $y_2(t)$ is a solution to the original, non-homogeneous, differential equation

$$y' + p(t)y = g(t).$$