DIFFERENTIAL EQUATIONS HOMEWORK 7
ADDITIONAL PROBLEMS

2006 SPRING

(1) In this problem we will consider the linear independence of solutions to the linear (homogeneous, constant coefficient) system of (first-order) differential equations

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}$$

(a) Suppose that A is a 2×2 matrix, and that λ_1 and λ_2 are its eigenvalues. (λ_1 and λ_2 may or may not be distinct.) Let $\mathbf{v}_1$ and $\mathbf{v}_2$ (respectively) be corresponding eigenvectors. Let

$$\mathbf{x}_1 = e^{\lambda_1 t}\mathbf{v}_1,$$

and

$$\mathbf{x}_2 = e^{\lambda_2 t}\mathbf{v}_2.$$

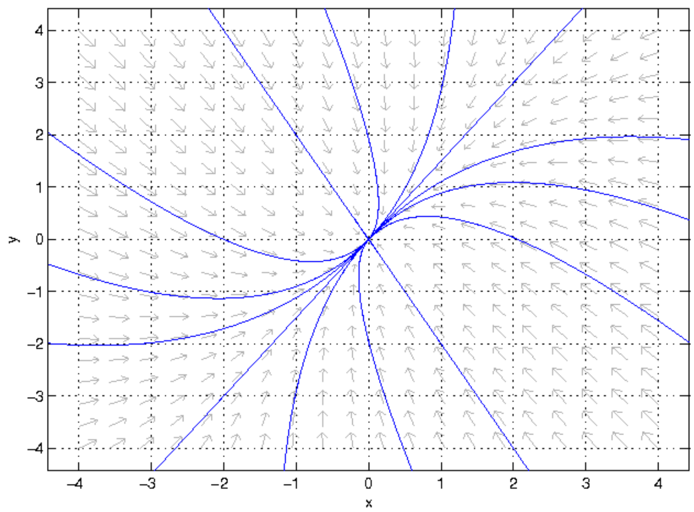
Show that $W[\mathbf{x}_1, \mathbf{x}_2](t)$ is non-zero for all t if and only if $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a linearly independent set.

(b) If A is an 3×3 matrix, suppose that $\lambda_1, \ldots, \lambda_3$ are its eigenvalues and $\mathbf{v}_1, \ldots, \mathbf{v}_3$ (respectively) be corresponding eigenvectors. Let

$$\mathbf{x}_i = e^{\lambda_i t}\mathbf{v}_i,$$

for $i = 1, \ldots, 3$. Show that $W[\mathbf{x}_1, \ldots, \mathbf{x}_3](t)$ is non-zero for all t if and only if $\{\mathbf{v}_1, \ldots, \mathbf{v}_3\}$ is a linearly independent set.

(2) Shown below is a phase portrait for a system of differential equations of the form $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$.



What are the eigenvectors of A? Determine two eigenvectors that have distinct eigenvalues. What sign do the eigenvalues corresponding to these eigenvectors have? What can be said about the relative size of the eigenvalues?