DIFFERENTIAL EQUATIONS HOMEWORK 4 ADDITIONAL PROBLEMS

2006 SPRING

(1) Consider the initial value problem
$$\frac{dx}{dt} = -\frac{1}{tx}, \quad x(-1) = 1$$
   (a) Use an analytic method to find an explicit solution for this problem. What is the domain of definition for this solution.
   (b) Use the Euler's method solver in the program `dfield` to compute (and graph) a numerical solution for this initial value problem. Use a step size of $\Delta t = .3$. How well does the numerical solution agree with the analytic solution for $t \geq 0$? For $t < -3/2$? What is happening here?
   (c) Repeat part 1b using step sizes $\Delta t = .03$ and $\Delta t = .003$. Does choosing a smaller step size change the behavior of the numerical solutions?
   (d) Is there any way to eliminate this strange behavior by choosing a sufficiently small step size?

(2) Consider the differential equation
$$\frac{dx}{dt} = e^t \cos(x).$$
   (a) Find all the equilibrium solutions to this differential equation. Where in the tx-plane is the Existence and Uniqueness Theorem satisfied?
   (b) Suppose that $\phi(t)$ is a solution with $-\frac{\pi}{2} < \phi(0) < \frac{\pi}{2}$. Can $\phi(t) > \frac{\pi}{2}$ for any value of t? Why or why not?
   (c) Use the Euler's method solver in the program `dfield` to compute (and graph) a numerical solution for this differential equation satisfying the initial condition $y(0) = 0$. Use a step size of $\Delta t = 0.1$. Something interesting happens. What? (If you don't see anything interesting, expand the size of your plot.)
   (d) Choose some smaller step sizes, and repeat part 2c. Does this solve the problem? Why not?

(3) Consider the system of differential equations

$$\frac{dx}{dt} = 2xy$$
$$\frac{dy}{dt} = -3y$$

Notice that the second differential equation does not depend on x. This is an example of a "partially decoupled" system.

(a) Find the solution for the second equation satisfying the initial condition $y(0) = 1$.

(b) Substitute the solution found in 3a for y in the first equation. Solve the resulting equation subject to the initial condition $x(0) = 2$.

(c) Find a solution to the system above that satisfies the initial condition $(x(0), y(0)) = (2, 1)$. [Hint: This should be easy once you complete 3a and 3b.]