21-260 **Differential Equations** **D. Handron**

Exam #1 Review

Closed book and notes; calculators not permitted. Be sure to show all work and explain your reasoning as clearly as possible.

1. Consider the initial value problem

$$xy' + 3y = x^3, \qquad y(1) = 10$$

   (a) Find the general solution to the differential equation.

   (b) Find the particular solution to the initial value problem.

2. (a) Use isoclines to draw the direction field for the differential equation

$$y' = \frac{1}{4}x^2 + y^2 - 1.$$

   Sketch the solution curve passing through the point $(0, 0)$.

   (b) Sketch the direction field for the differential equation $y' = (y-1)(y+1)(y+2)^2$. Are there any constant solutions? Why might you think so? How can you be certain?

3. A system consists of three tanks containing salt solutions. The first tank holds 100ℓ of solution, the second 100ℓ and the third initially holds 50ℓ of solution. Tank 1 initially contains 25g of salt in solution, Tank 2 contains 10g initially, and Tank 3 begins with 50g.

   Pure water is added to the first tank at a rate of 5ℓ per minute. Two spigots allow the water to flow from Vat 1 to Vats 2 and 3. The rate of flow for these spigots is 4ℓ per minute and 1ℓ per minute respectively. The solution flows from Vat 2 to Vat 3 at a rate of 4ℓ per minute. The solution from Vat 3 is allowed to flow onto the ground at a rate of 6ℓ per minute (most likely destroying a fragile ecosystem, but that is none of our concern).

   Let $x_j(t)$ be the ammount of salt in tank j at time t.

   (a) Write three differential equations that describes the behovior of x_1, x_2 and x_3.

   (b) Verify that $x_1(t) = 25e^{-t/20}$.

   (c) Find the solution for $x_2(t)$.

4. Consider the differential equation

$$\frac{dy}{dx} = (y-1)(y+2)^2(y^2+1).$$

   (a) Draw the direction field for this differential equation.

   (b) Let $y(t)$ be the solution satisfying the initial condition $y(0) = 0$. Can the value of $y(t)$ ever be less than -2? Why or why not?

5. Consider the differential equation

$$\frac{dy}{dt} = y + e^t$$

   This is a linear differential equation. Find the solution satisfying the initial condition $y(1) = 1$.