21-260 **Differential Equations** **D. Handron**

Review #3

1. Consider the linear system of differential equations

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x},$$

where

$$A = \begin{bmatrix} 1 & 5 \\ 10 & -4 \end{bmatrix}.$$

   (a) Find the eigenvalues of A.

   (b) Find the eigenvectors of A.

   (c) Solve the initial value problem $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$, $\mathbf{x}(0) = \begin{bmatrix} 0 \\ 9 \end{bmatrix}$.

2. Let

$$B = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

The real number 2 is an eigenvalue of B with multiplicity 3, and $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ is an eigenvector. There are no other linearly independent eigenvectors.

   (a) Find a chain of generalized eigenvectors of B with length 3.

   (b) Find 3 linearly independent solutions to $\frac{d\mathbf{x}}{dt} = B\mathbf{x}$.

   (c) Find the general solution to $\frac{d\mathbf{x}}{dt} = B\mathbf{x}$.

3. A mass-spring-dashpot system can be modeled by the second order equation

$$m\frac{d^2x}{dt^2} = -k_s\frac{dx}{dt} - k_d x,$$

where m is the mass, k_s is the spring constant and k_d is the damping coefficient.

   (a) A certain system of this type with $m = 1$ can also be modeled by the first order system

$$\begin{bmatrix} x' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -4 \end{bmatrix}\begin{bmatrix} x \\ v \end{bmatrix}.$$

   What is the spring constant in this system? What is the damping coefficient?

(b) The complex number $-2+i$ is an eigenvalue of the matrix $\begin{bmatrix} 0 & 1 \\ -5 & -4 \end{bmatrix}$. An eigenvector corresponding to $-2+i$ is $\begin{bmatrix} 1 \\ -2+i \end{bmatrix}$. What is the general solution?

(c) What is the general solution to the second order equation $\frac{d^2x}{dt^2} = -4\frac{dx}{dt} - 5x$?

4. Consider the system of differential equations,

$$\begin{aligned} \frac{dx}{dt} &= 3x - xy \\ \frac{dy}{dt} &= -4y + 2xy. \end{aligned}$$

(a) Find the equilibrium points of this system.

(b) Classify the equilibrium point at $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ as to type and stability.