21-260 **Differential Equations** **D. Handron**

Review Exam #1

Closed book and notes; calculators not permitted. Be sure to show all work and explain your reasoning as clearly as possible. Good Luck.

1. A system of vats and pipes is constructed in the following way:

   Vat 1 initially contains 100 ℓ of water and 25g of salt. Vat 2 initially contains 100 ℓ of water and 10g of salt. Vat 2 initially contains 50 ℓ of water and 50g of salt.

   Pure water is added to Vat 1 at a rate of 5ℓ per minute. Two spigots allow the solution to flow from Vat 1 to Vats 2 at a rate of 4ℓ per minute and to Vat 3 at a rate of 1ℓ per minute. The solution flows from Vat 2 to Vat 3 at a rate of 4ℓ per minute. Solution from Vat 3 is allowed to flow onto the ground at a rate of 6ℓ per minute (most likely destroying a fragile ecosystem, but that is none of our concern).

   (a) Write a system of equations that describes x_1, x_2 and x_3. (It may help to draw a diagram.)

   (b) Verify that $x_1(t) = 25e^{-t/20}$.

   (c) Find the solution for $x_2(t)$.

2. Consider the partially decoupled system

$$\frac{dx}{dt} = tx$$

$$\frac{dy}{dt} = txy.$$

   Solve the initial value problem where $x(0) = 1$ and $y(0) = e$.

3. Compute the Laplace transform of the function

$$f(t) = e^{at}g(t), \quad \text{where} \quad g(t) = \begin{cases} 2t & 0 \le t < 3 \\ 0 & t \ge 3 \end{cases}$$

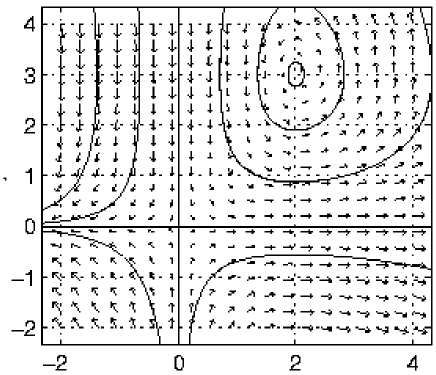
4. Use Laplace transforms to solve the initial value problem

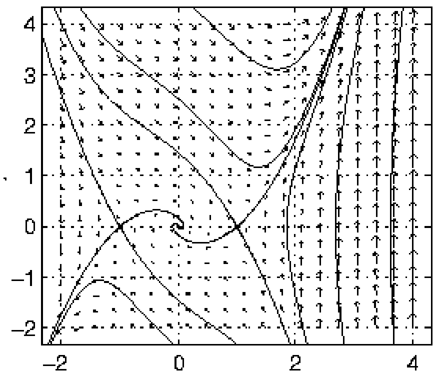
$$x'' + 4x = 1, \quad \text{where} \quad x(0) = 1, \text{ and } x'(0) = 0.$$

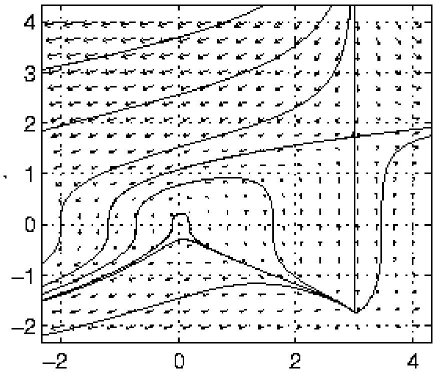
5. Consider the system of differential equations,

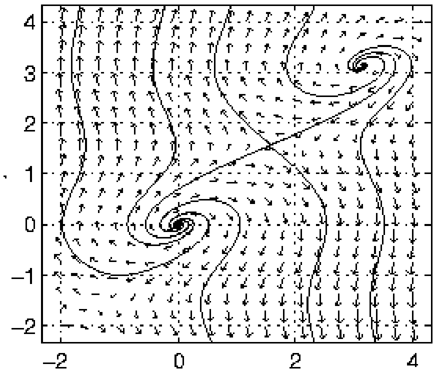
$$\begin{aligned} \frac{dx}{dt} &= 3x - xy \\ \frac{dy}{dt} &= -4y + 2xy. \end{aligned}$$

(a) Circle the phase plane for this system:









(b) Find the equilibrium points of this system.