21-260 **Differential Equations** **D. Handron**

Review Exam #1

Closed book and notes; calculators not permitted. Be sure to show all work and explain your reasoning as clearly as possible. Good Luck.

1. Consider the first order differential equation

$$\frac{dy}{dt} = \frac{t}{2y+1}$$

   (a) Find the general solution.

   (b) Find the particular solution with $y(1) = \frac{3}{2}$.

   (c) Is this the unique solution with $y(1) = \frac{3}{2}$? How can you tell?

2. (a) Solve the initial value problem

$$xy' + 3y = x^3, \qquad y(1) = 10$$

   (b) What can you say about the existence and uniqueness of a solution to the above differential equation with initial condition $y(3) = 2$? What about $y(0) = 1$?

3. (a) Use isoclines to draw the slope field for the differential equation

$$y' = \frac{1}{4}x^2 + y^2 - 1.$$

   Sketch the solution curve passing through the point $(0, 0)$.

   (b) Sketch the phase line for the differential equation $y' = (y-1)(y+1)(y+2)^2$ Label each critical point as stable, semi-stable or unstable.

4. An undamped, forced harmonic harmonic oscillator may be modelled by the equation $mx'' + kx = f(t)$, where m is the mass of the weight, k is the spring constant, and $f(t)$ is the force applied to the mass. Consider the case where $m = 2$, $k = 8$ and $f(t) = 3\cos(\omega t)$.

   (a) Use the method of undetermined coefficients to find a particular solution, $x_p(t)$.

   (b) For what value of ω will resonance occur? How can this be determined from the particular solution found in part (a)?

5. Compute the Laplace transform of the function

$$f(t) = e^{at}g(t), \quad \text{where} \quad g(t) = \begin{cases} 2t & 0 \le t < 3 \\ 0 & t \ge 3 \end{cases}$$