1. (25 points)

(a) Find all solutions to the system
\[
\begin{align*}
x + 2y - 2z - 8w &= 0 \\
2x + 4y - 3z - 13w &= 0 \\
-x - 2y + 3z + 11w &= 0
\end{align*}
\]

Describe the nullspace of the matrix \( A = \begin{bmatrix}
1 & 2 & -2 & -8 \\
2 & 4 & -3 & -13 \\
-1 & -2 & 3 & 11
\end{bmatrix} \) as a span of vectors.

(b) Is the vector \( b = \begin{bmatrix}
-8 \\
-13 \\
11
\end{bmatrix} \) in the column space of the matrix \( B = \begin{bmatrix}
1 & 2 & -2 \\
2 & 4 & -3 \\
-1 & -2 & 3
\end{bmatrix} \)? Why or why not?

2. (25 points)

(a) Find the \( LU \) factorization for the matrix
\[
M = \begin{bmatrix}
-1 & 4 & 1 \\
2 & -6 & 5 \\
-1 & 6 & 6
\end{bmatrix}
\]
and solve the system \( Mx = \begin{bmatrix}
0 \\
-5 \\
-3
\end{bmatrix} \) as two triangular systems.

(b) Use Gauss-Jordan elimination to find the inverse of the matrix
\[
N = \begin{bmatrix}
2 & 1 \\
-1 & 2
\end{bmatrix}
\]

3. (25 points)

(a) Find a system of equations, with two equations and three unknowns, that has solutions
\[
\begin{bmatrix}
\begin{array}{c}
x_1 \\
x_2 \\
x_3
\end{array}
\end{bmatrix} = \begin{bmatrix}
4 \\
5 \\
0
\end{bmatrix} + t \begin{bmatrix}
3 \\
2 \\
1
\end{bmatrix}.
\]

You may write the system in matrix form \( Ax = b \).
(b) Find a system, with three equations and three unknowns, having the same solutions as in part (a) and with $b_3 = b_2 - b_1$.

4. **(25 points)** Find conditions on $a$, $b$, $c$, $d$, $e$, $f$, and $g$ that ensure the matrix

$$
\begin{bmatrix}
a & e & 0 & 0 \\
a & b & f & 0 \\
a & b & c & g \\
a & b & c & d
\end{bmatrix}
$$

is invertible.