MATRIX ALGEBRA WEEK #12
ADDITIONAL PROBLEMS

2005 SPRING

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = 2\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad T\left(\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}\right) = -\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix},$$

and

$$T\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = 3\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

The following problems refer to the linear transformation T.

(1) Find a basis $\mathcal{B}$ for $\mathbb{R}^3$ such that the $\mathcal{B}$-coordinate matrix $[T]_{\mathcal{B}}$ of the transformation T is diagonal.
(2) Let A be the standard matrix for the linear transformation T. Find a diagonalization of A. (You shouldn't need to compute the matrix A to complete this problem.)
(3) Find the standard matrix A for the transformation T.