

## 21-124 MODELING WITH DIFFERENTIAL EQUATIONS

### HOMEWORK 2: DUE IN CLASS ON FEBRUARY 12

1. In this problem, we will consider the initial value problem

$$\frac{dy}{dt} = (1 + y^2) \cos(t); \quad y(0) = 0.$$

- Find the exact solution (analytically).
  - Use the MATLAB routine `eul` to produce an approximate solution on the interval  $[0, 6]$ , with step size  $h=1$ . Determine the maximum error for this approximation by comparing with the exact solution. Repeat this eight times, halving the step size each time. Produce a log-log plot showing maximum error vs. step size.
  - Repeat part 1b using `rk2` instead of `eul`.
  - Repeat part 1b using `rk4` instead of `eul`.
  - Use Matlab to display on a single diagram the plots of the exact solution and the approximate solutions generated by each of the three methods using a step size of  $h=.25$ . Use a distinctive marking for each method. (Note that when you print the graph it will appear in black and white, so simply choosing different colors may not work very well.)
2. A function of the form  $y = ax^b$  is called a power function. Use Matlab's `plot` command to sketch the plot of each of the power functions on the interval  $[0, 2]$ :
- $y = 2x^3$
  - $y = 200x^4$
  - $y = 50x^{-2}$

Then use the `loglog` command to produce a log-log plot on the same interval.

Describe what the graph of a power function looks like when drawn on a `loglog` graph.

3. The accuracy of any numerical method in solving a differential equation of the form  $y' = f(t, y)$  depends on how strongly the equation depends on the variable  $y$ . (The error bounds depend on the derivatives of  $f$  with respect to  $y$ .) To see this experimentally, consider the two initial value problems

$$y' = y; \quad y(0) = 1$$

and

$$y' = e^t; \quad y(0) = 1.$$

Note that  $y(t) = e^t$  is the solution to both problems.

- Use `eul` to compute approximate solutions to the two initial value problems on  $[0, 1]$  using a step size of  $h=.01$ . Compare the accuracy of the solutions to the two problems.
- Repeat this process for the routines `rk2` and `rk4`.