

21-124 MODELING WITH DIFFERENTIAL EQUATIONS

HOMEWORK 1

1. Suppose a population of rabbits on an island grows according to the exponential growth model, with a growth rate of α . Now imagine that hunting is allowed to take place on the island. Naturally, some seasons will be more conducive to hunting than others. The bitter winter cold will drive off all but the most hardy of hunters, while the warm summer sun will attract them in droves. Since the change of seasons is a more or less continuous process, you can assume that the rate of hunting will change continuously.

Derive a mathematical model corresponding to these assumptions. Describe any additional assumptions you make regarding the nature of the hunting. Be sure to include a parameter describing the “effectiveness” of the hunters. Use `dfield` to produce some solution curves for various starting populations and degrees of effectiveness. Try to show as many different types of behavior as possible

Describe the effect of this hunting on the growth of the rabbit population? Under what conditions will they continue to grow without bound? Under what conditions will they go extinct? Is there a situation in which the population seems to stabilize in some range?

2. Two tanks hold the same volume of water, and have holes in the bottom with the same area. One is tall and skinny the other is short and fat. Which will drain faster? Illustrate your result with two examples.
3. Two tanks are the same height, hold the same volume of water, and have holes in the bottom with the same area. One is skinny at the bottom and fat at the top, the other is fat at the bottom and skinny at the top. Which will drain faster? Illustrate your result with two examples.