21-122 Integration, Differential Equations, and Approximation D. Handron

Final Exam Review Fall 2007

Note: You may notice that there are no problems on this reveiw sheet that involve second order differential equations. You should look to the problems in the textbook to study that material. I simply have not covered that portion of the course at this level before, so I don't have any old exam problems to give you.

1. Determine the interval of convergence and the radius of convergence for the power series $\sum_{n=1}^{\infty}(-1)^n \frac{(2x+3)^n}{n\ln(n)}$.

2. (a) Find the Taylor series for $f(x) = \ln(x)$ around $x = 2$.
   (b) Find the Taylor series for $g(x) = (x-2)^2 \ln(x)$ around $x = 2$.

3. Recall that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$.
   (a) Use this MacLauren series to find a Taylor series for $-\ln(1-x)$.
   (b) Find the first five terms in the MacLauren series for $-\frac{\ln(1-x)}{1-x}$.

4. Recall that

$$e^x = \sum_{n=1}^{\infty} \frac{x^n}{n!} \qquad \frac{1}{1-x} = \sum_{n=1}^{\infty} x \qquad \cos(x) = \sum_{n=1}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$$

   (a) Express $\int \cos(x^2)dx$ as a power series.
   (b) What is the radius of convergence for this power series?
   (c) Find the fourth degree Taylor polynomial for $\frac{e^x}{1-x}$.

5. (a) Find the Taylor series of $(2-x)^{-3}$ centered at 1.
   (b) Use Taylor's Inequality to find an upper bound for $R_3(x)$, when $\frac{1}{2} \le x \le \frac{3}{2}$.

6. Determine whether the series

$$\sum_{n=0}^{\infty} \left(\frac{n}{n^2+1}\right)^n$$

   converges absolutely, converges conditionally, or diverges.

7. Find the radius of convergence and interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(x-2)^n}{n(\ln n)^2}.$$

8. (a) Determine the power series centered at 0 which represents the function $\sinh(x) = \frac{e^x - e^{-x}}{2}$.
   (b) Determine the power series centered at 0 which represents the function $\cosh(x) = \frac{d}{dx}\sinh(x)$