Final Exam Reference Table

I. Trigonometric Identities

1. \( \tan^2 \theta + 1 = \sec^2 \theta \)
2. \( \cot^2 \theta + 1 = \csc^2 \theta \)
3. \( \sin^2 \theta = \frac{1}{2} [1 - \cos(2\theta)] \)
4. \( \cos^2 \theta = \frac{1}{2} [1 + \cos(2\theta)] \)
5. \( \sin \theta \cos \theta = \frac{1}{2} \sin(2\theta) \)
6. \( \sin A \cos B = \frac{1}{2} [\sin(A - B) + \sin(A + B)] \)
7. \( \sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)] \)
8. \( \cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)] \)

II. Error Estimates for Numerical Integration

The expressions below give an upper bound for approximations to \( \int_a^b f(x) \, dx \) using the trapezoid rule, the midpoint rule, and Simpson’s rule. In the expressions below \( K \) is a number such that \( |f''(x)| \leq K \) for \( a \leq x \leq b \) and \( M \) is a number such that \( |f^{(4)}(x)| \leq M \) for \( a \leq x \leq b \).

The number \( n \) represents the number of subintervals into which \([a, b]\) is divided.

\[
|E_T| \leq \frac{K(b - a)^3}{12n^2}
\]
\[
|E_M| \leq \frac{K(b - a)^3}{24n^2}
\]
\[
|E_S| \leq \frac{M(b - a)^5}{180n^4}
\]

III. Remainder Estimates for Series and Taylor’s Formula

If \( \sum_{n=1}^{\infty} (-1)^n b_n \) converges by the alternating series test, then \( |R_n| = |s - s_n| \leq |b_{n+1}| \).

If \( \sum_{n=1}^{\infty} a_n \) converges by the integral test, using \( f(x) \), then \( \int_{n+1}^{\infty} f(x) \, dx \leq R_n \leq \int_n^{\infty} f(x) \, dx \).

Taylor’s Formula: \( R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!} (x - a)^{n+1} \) for some \( z \) between \( a \) and \( x \).