21-122 Integration, Differential Equations, and Approximation D. Handron

Exam #2 Review Fall 2007

1. Find the length of the curve $y = \ln(\sec(x))$ from $x = 0$ to $x = \frac{\pi}{4}$.

2. Find the length of the curve
$$y = \frac{x^3}{6} + \frac{1}{2x}$$
between the points $(\frac{1}{2}, \frac{31}{24})$ and $(1, \frac{2}{3})$.

3. (a) Sketch the direction field for $\frac{dy}{dt} = y - t$ in the region where $0 \le x \le 4$ and $0 \le y \le 4$.

4. Find the general solution to the differential equation
$$\frac{dy}{dt} = (y^2 + 1)(t + t^2).$$

5. You have a bank account that earns 4% interest per year. You earn \$4500 per year designing web pages for local businesses, and you spend \$5000 buying CD's and computer games.
   (a) Give a differential equation that models your account balance t years from now, $A(t)$.
   (b) Draw the phase line for this differential equation. How much money must you have in your account today in order to support your lifestyle indefinitely.
   (c) If $A(0) = 10,000$, how much money will be in the account in 5 years? (It may help to know that $e^{1/5} \approx \frac{6}{5}$.)

6. A man comes to your chemistry lab with a bear skin he claims was worn by Ghingis Khan when he conquered the Persians in 1221 AD. Upon examining the skin, you find it contains 95% as much ^{14}C as does animal material on the earth today. ^{14}C has a half-life of 5730 years. Was the man correct?

   The following relations may be helpful:
$$\ln\left(\frac{1}{2}\right) \approx -\frac{7}{10}, \quad e^{-\frac{7}{57300}779} \approx .909, \quad \frac{57300}{7}\ln(.95) \approx -420.$$

7. (a) Find the general solution to the differential equation $\frac{dy}{dt} = y\cos(t)$.

(b) Find the particular solution that satisfies the initial condition $y(0) = 1$. Note that for this solution, $y(\frac{\pi}{2}) = e \approx 2.718$.

8. The size of a population of rabbits on a small island is modeled by the logistic equation

$$\frac{dP}{dt} = 5P\left(1 - \frac{P}{1000}\right).$$

where P denotes the number of rabbits, and t is time measured in years.

(a) Sketch the phase line and slope field for this differential equation.

(b) On the slope field above, sketch the solution satisfying $P(0) = 200$. What is $\lim_{t\to\infty} P(t)$?

(c) Suppose that the island is stocked with 100 additional rabbits each year. Modify the differential equation to account for this additional assumption.

(d) What are the equilibrium points of the modified system? If $P(0) = 0$, what is $\lim_{t\to\infty} P(t)$? It might help to know that $\sqrt{27} \approx 5.2$ and you might want to consider the phase line for the modified system.