21-122 Integration, Differential Equations and Approximation D. Handron

Exam #1 Review Fall 2006

1. (a) Approximate $\int_{-2.5}^{2.5} x^2 dx$ using the Midpoint Rule with $n = 5$.
   (b) What was the actual error in your approximation? (It may help to know that $\int_{-2.5}^{2.5} x^2 dx = \frac{125}{12}$.)
   (c) What value of n must you choose to be assured the same accuracy using the Trapezoid Rule. (It may help to know that $\sqrt{50} = 7.0711$.)

2. Compute the following integrals:

   (a)
$$\int x^3\sqrt{9 - 4x^2}dx$$

   (b)
$$\int \frac{5x^2 - 6x + 2}{(x-2)(x^2+1)}$$

3. (a) Does $\int_2^6 \frac{dx}{\sqrt{x-2}}$ converge? If so, evaluate it.
   (b) Does $\int_6^\infty \frac{dx}{\sqrt{x-2}}$ converge? If so, evaluate it.
   (c) Does $\int_2^\infty \frac{dx}{\sqrt{x-2}}$ converge? If so, evaluate it. If not, why not?

4. Compute the integral
$$\int \frac{\sin(\sqrt{x+5})\sin(2\sqrt{x+5})}{4\sqrt{x+5}}dx.$$

5. Compute the indefinite integral
$$\int \frac{3x^2 - 4x + 5}{(x-1)(x^2+1)}.$$

6. Compute the indefinite integral
$$\int \frac{dx}{x^4\sqrt{16x^2 - 9}}.$$

   You may leave your answer in terms of powers of 3 and 4, you don't need to multiply them out.

7. (a) Show that in approximating the value of $\int_a^b f(x)dx$, the Trapezoid Rule and the Midpoint Rule satisfy

$$\frac{1}{2}(T_n + M_n) = T_{2n}.$$

(b) Approximate the integral $\int_0^{3\pi} \sin(x)dx$ using Simpson's Rule with $n = 6$, i.e. compute S_6. It may help to know that $\frac{2\pi}{3} \approx 2.1$.

(c) The exact value of the integral above is $\int_0^{3\pi} \sin(x)dx = 2$. How large must n be for the Trapezoid rule approximation T_n to satisfy $|E_T| \leq (2 - S_n)$? It may help to know that $\frac{10(3\pi)^3}{12} \approx 697.64$ and $\sqrt{697.64} \approx 26.41$.

8. Does the integral

$$\int_1^\infty \frac{\sin^2(x)}{x^2 + \sqrt{x}} dx$$

converge or diverge?

9. Does the integral

$$\int_1^\infty \frac{3dx}{2x\sqrt{4 + 5x}}$$

converge or diverge? If it converges, what is it's value? It may help to know that $\lim_{t\to\infty} \frac{\sqrt{4+5t}-2}{\sqrt{4+5t}+2} = 1$.

10. Find the length of the curve $y = \ln(\sec(x))$ from $x = 0$ to $x = \frac{\pi}{4}$.

11. Find the length of the curve

$$y = \frac{x^3}{6} + \frac{1}{2x}$$

between the points $(\frac{1}{2}, \frac{31}{24})$ and $(1, \frac{2}{3})$.