21-118 **Calculus of Approximation** **D. Handron**

Exam #1
February 11, 2000 **Name:**______________________________

Closed book and notes; calculators not permitted.

Write your solutions in the space below each problem. Be sure to show all work and explain your reasoning as clearly as possible. You may use the back of the page if you need more space; please indicate clearly if you do so. Good Luck.

Problem	**Points**	**Score**
1	25	
2	25	
3	25	
4	25	
Total	100	

1. **(25 points)** Newton's method may be used to solve the equation

$$(x+8)^2 = 0$$

Begin with $x_1 = 8$, and compute x_2, x_3, x_4 and x_5.

2. **(25 points)** The limits below all represent indeterminate forms. For each one, determine the type (e.g. $\frac{0}{0}$ or $0 \cdot \infty$) and evaluate the limit.

(a)

$$\lim_{x \to 0} \frac{\cos(x) - 1}{x^2}$$

(b)

$$\lim_{x \to 1} \left(\frac{1}{x-1} \right) \ln(x)$$

(c)

$$\lim_{x \to \infty} \left(\frac{1}{x} \right)^{\left(\frac{1}{x} \right)}$$

3. **(25 points)**

(a) Let $a_n = \frac{n^2-1}{n^2}$. What is $\lim_{n\to\infty} a_n$? Does $\sum_{n=1}^{\infty} a_n$ converge? Why or why not?

(b) Let $b_n = \frac{1}{n\ln(n)}$. What is $\lim_{n\to\infty} b_n$? Does $\sum_{n=1}^{\infty} b_n$ converge? Why or why not?

(c) Let $t_n = \frac{(n!)^2}{(2n)!}$. Is the sequence $\{t_n\}$ monotonic (increasing or decreasing)? (Hint: what can you say about t_n/t_{n+1}?)

(d) Is the sequence $\{t_n\}$ bounded? Does the limit $\lim_{n\to\infty} t_n$ exist? Why or why not?

4. **(25 points)** The **Sierpinski triangle** is constructed by removing the center one-fourth of an equilateral triangle with area 1, then removing the centers of three smaller remaining triangles, and so on. Show that the sum of the sreas of the removed triangles is 1.