

Measure Theory and Lebesgue Integration.

Gautam Iyer, Fall 2013

- L1, Mon 8/26.* • Introduction and motivation.
• Lebesgue Measure on \mathbb{R}^n .
– [1, §13] Abstract σ -algebras and measures.
- L2, Wed 8/28.* – [1, §11] Volumes of cells and intervals.
- L3, Fri 8/30.* – [1, §12] Lebesgue outer measure.
- L4, Wed 9/04.* – [1, §13; 2, §1.3] The Caratheodory condition.
- L5, Fri 9/06.* – [1, §13] The Lebesgue σ -algebra and uniqueness.
- L6, Mon 9/9.* • Abstract measures
– [2, §1.6] π -systems and λ -systems.
- L7, Wed 9/11.* – [2, §1.5] Regularity of measures.
- L8, Fri 9/13.* – [2, §1.4] Non Lebesgue-measurable sets
- L9, Mon 9/16.* – [2, §1.5] Completion
- L10, Wed 9/18.* • Measurable functions
– [2, §2.1] Definition and elementary properties
- L11, Fri 9/20.* – [2, §2.1] Cantor function & non-Borel sets.
- L12, Mon 9/23.* – [2, §2.1] Approximation results.
- L13, Wed 9/25.* • Integration.
– [2, §2.3; 3, §2.2] Integrating non-negative functions.
- L14, Fri 9/27.* – [2, §2.4; 3, §2.2] Monotone convergence and linearity.
- L15, Mon 9/30.* – [2, §2.4; 3, §2.2] Fatou's lemma and dominated convergence.
- L16, Wed 10/2.* • Convergence
– [2, §3.1] Convergence almost everywhere and in measure.
- L17, Fri 10/4.* – L^p spaces.
* [2, §3.2] Normed vector spaces and Banach spaces.
- L18, Mon 10/7.* * [5, §3] Hölder's inequality and duality.
- L19, Wed 10/9.* * [2, §3.3; 5, §3] Completeness and Convergence.
- L20, Mon 10/14.* – Uniform integrability, Vitali convergence theorem.
- L21, Wed 10/16.* – Conditions for uniform integrability.
- L22, Mon 10/21.* • Signed Measures
– [2, §4.1] Hanh and Jordan decompositions.
- L23, Wed 10/23.* – [2, §4.2] Radon Nikodym theorem.
- L24, Fri 10/25.* – [2, §4.3] Lebesgue Decomposition

- L25, Mon 10/28.* – [2, §3.5, §4.5] The dual of L^p .
- L26, Wed 10/30.* • Integration on Product Spaces.
– [2, §5.1] Product σ -algebras.
- L27, Fri 11/1.* – [2, §5.2] Fubini and Tonelli theorems.
- L28, Mon 11/4.* – [2, §5.3] Distribution functions and applications.
- L29, Wed 11/6.* • Convolutions
– [5, §7] Young's inequality
– [5, §7] L^p -convergence of approximate identities.
- L30, Thu 11/7.* • Fourier Series.
– [6, §1] Dirichlet, Fejér kernels and convergence of Cesàro sums.
- L31, Mon 11/11.* – [6, §1] Riemann Lebesgue Lemma Parseval's identity
- L32, Wed 11/13.* – Periodic Sobolev spaces and embedding theorems.
- L33, Fri 11/15.* • Lebesgue Differentiation
– [2, §6.2; 5, §7] Vitali covering and the Maximal function.
- L34, Mon 11/18.* – [2, §6.2; 5, §7] Lebesgue points and differentiation of measures.
- L35, Wed 11/20.* – [2, §6.3; 5, §7] Absolute continuity of functions of one variable.
- L36, Fri 11/22.* – [2, §6.3; 5, §7] Fundamental theorem of Calculus.
- L37, Mon 11/25.* – [2, §6.1; 5, §7] Change of variable.
- L38, Mon 12/2.* • Fourier Transform
– [3, §8.3; 5, §9] L^1 theory and inversion.
- L39, Wed 12/4.* – [3, §8.3; 5, §9] Plancherel theorem and L^2 -theory.
- L40, Fri 12/6.* – [3, §9.3] Sobolev spaces and embeddings.

References

- [1] R. G. Bartle, *The elements of integration and Lebesgue measure*, Wiley Classics Library, John Wiley & Sons Inc., New York, 1995. Containing a corrected reprint of the 1966 original [*The elements of integration*, Wiley, New York; MR0200398 (34 #293)]; A Wiley-Interscience Publication. MR1312157 (95k:28001)
- [2] D. L. Cohn, *Measure theory*, Birkhäuser Boston, Mass., 1980. MR578344 (81k:28001)
- [3] G. B. Folland, *Real analysis*, 2nd ed., Pure and Applied Mathematics (New York), John Wiley & Sons Inc., New York, 1999. Modern techniques and their applications; A Wiley-Interscience Publication. MR1681462 (2000c:00001)
- [4] H. L. Royden, *Real analysis*, 3rd ed., Macmillan Publishing Company, New York, 1988. MR1013117 (90g:00004)
- [5] W. Rudin, *Real and complex analysis*, 3rd ed., McGraw-Hill Book Co., New York, 1987. MR924157 (88k:00002)
- [6] W. Schlag, *Lecture notes on Harmonic Analysis*, available at http://www.math.uchicago.edu/~schlag/harmonicnotes_old.pdf.