1. Let $f$ be a continuous $2\pi$ periodic function, and define $g(x) = e^{ix}f(x)$. Let $(c_n)$ be the complex Fourier coefficients of $f$ and $(d_n)$ be the complex Fourier coefficients of $g$. Given that $c_n = (1 + |n|)^{-372}$ for all $n \in \mathbb{Z}$, find $d_n$ explicitly.

2. Consider the PDE $\partial_t u + 2\partial_x u - \partial_x^2 u = 0$ for $x \in (0, \pi)$, $t > 0$, with boundary conditions $u(0, t) = 0 = u(\pi, t)$. Use separation of variables to write down the general solution to this PDE as an infinite series. [You may buy the general solution of the ODE $y'' + by' + cy = 0$ for 2 points. In this case this problem will be scored out of 3 and not 5.]

3. Find an explicit formula for a function $u$ that satisfies the heat equation $\partial_t u - \frac{1}{2} \partial_x^2 u = 0$ for $x \in \mathbb{R}$, $t > 0$, with initial data $u(x, 0) = \text{sign}(x)$ and boundary conditions

$$
\lim_{x \to \infty} u(x, t) = 1, \quad \text{and} \quad \lim_{x \to -\infty} u(x, t) = -1.
$$

You may leave your answer as an unsimplified integral, provided the integrand is an explicit function.

4. Let $L, T > 0$ and $a, c$ be two functions such that $a(x, t) \geq 0$ and $c(x, t) \geq 0$. Suppose $u$ is twice differentiable on the open rectangle $R \triangleq (0, L) \times (0, T)$ and satisfies the partial differential inequality

$$
\partial_t u - a\partial_x^2 u + cu \leq 0.
$$

Suppose further $u$ is continuous on the closed rectangle $\bar{R} \triangleq [0, L] \times [0, T]$. If $M$ is the maximum of $u$ in $\bar{R}$, and $M \geq 0$, then show that $u$ attains the value $M$ on the sides or bottom of $\bar{R}$. [A more general version of this question was on Homework 4. Please provide a complete proof here without quoting and using the result from homework 4.]