1. Find the general solution of the PDE
\[ \partial_x u + x\partial_y u = x^2 + y. \]

2. Suppose \( u \) satisfies the PDE
\[ \partial_t^2 u + 3\partial_x \partial_t u - 2\partial_x^2 u = 0, \]
and as \( x \to \pm \infty \) we have \( u \to 0, \partial_t u \to 0 \) and \( \partial_x u \to 0 \). Find a constant \( \alpha \in \mathbb{R} \) so that the energy \( E \) defined by
\[ E(t) = \int_{-\infty}^{\infty} \left[ (\partial_t u(x,t))^2 + \alpha (\partial_x u(x,t))^2 \right] dx \]
is constant as a function of time.

3. Let \( u(x,t) \) be the population of a virus at the point \( x \in \mathbb{R}^3 \) and time \( t \). Suppose the virus population changes as follows:
   
   (i) Due to overcrowding, the virus migrates from regions of high population to regions of low population at a rate proportional to the gradient. Namely, the rate of migration in a particular direction \( v \) equals \( \kappa (\nabla u) \cdot v \), where \( \kappa > 0 \) is some constant.
   
   (ii) The rate at which the virus population grows (due to reproduction and death) equals \( u(1-u) \).

   Find a PDE satisfied by the function \( u \). [For half credit, you may instead do the one dimensional version of this question assuming \( x \in \mathbb{R} \).]

4. Let \( D \subseteq \mathbb{R}^3 \) be a sphere of radius 1. We claim that there exists only one real number \( \alpha \in \mathbb{R} \) so that the PDE
\[ -\Delta u = 1 \text{ in the domain } D \text{ with Neumann boundary conditions } \hat{n} \cdot \nabla u = \alpha \text{ on } \partial D \]
has a solution. Find \( \alpha \).