

Measure Theory and Lebesgue Integration.

Gautam Iyer, Fall 2012

- L1, Mon 8/27.* • Introduction
- Lebesgue Measure on \mathbb{R}^n .
- [1, §11] Volumes of cells and intervals.
- L2, Wed 8/29.* – [1, §12] Lebesgue outer measure.
- L3, Fri 8/31.* – [1, §13] Abstract σ -algebras and measures.
- L4, Wed 9/05.* – [2, §1.3] General outer measures.
- [1, §13] The Caratheodory condition.
- L5, Fri 9/07.* – [1, §13] Uniqueness of the Lebesgue measure
- [2, §1.2] Basic properties.
- L6, Mon 9/10.* – [2, §1.4] Regularity.
- [2, §1.4] Non Lebesgue-measurable sets
- Abstract measures
- [2, §1.5] Completion
 - [2, §1.5] Regularity
 - [2, §1.6] π -systems and λ -systems.
- L7, Wed 9/12.* – [2, §1.5] Completion
- L8, Fri 9/14.* – [2, §1.5] Regularity
- L9, Mon 9/17.* – [2, §1.6] π -systems and λ -systems.
- L10, Wed 9/19.* • Integration
- [2, §2.1] Measurable functions
- L11, Fri 09/21.* – [2, §2.1] Cantor function & non-Borel sets.
- L12, Mon 09/24.* – [2, §2.1] Simple functions
- L13, Wed 09/26.* – [4, §4.2] Integrating bounded functions on finite measure sets.
- L14, Fri 09/28.* – [4, §4.3] Integration of non-negative functions
- L15, Mon 10/01.* – [4, §4.4] The general integral.
- L16, Wed 10/03.* – [2, §2.6] Push forward of measures.
- Convergence
- [2, §3.1] Convergence almost everywhere and in measure.
 - [2, §3.2] Normed vector spaces and L^p .
 - [5, §3] Jensen and Hölder inequalities.
 - [2, §3.3; 5, §3] L^p -spaces, Completeness and Convergence.
 - Uniform integrability, Vitali convergence theorem.
 - Conditions for uniform integrability.
- L17, Fri 10/05.* – [2, §3.1] Convergence almost everywhere and in measure.
- L18, Mon 10/08.* – [2, §3.2] Normed vector spaces and L^p .
- L19, Wed 10/10.* – [5, §3] Jensen and Hölder inequalities.
- L20, Fri 10/12.* – [2, §3.3; 5, §3] L^p -spaces, Completeness and Convergence.
- L21, Mon 10/15.* – Uniform integrability, Vitali convergence theorem.
- L22, Mon 10/22.* – Conditions for uniform integrability.
- L23, Wed 10/24.* • Signed Measures
- [2, §4.1] Hanh and Jordan decompositions.
- L24, Fri 10/26.* – [2, §4.2] Radon Nikodym theorem.
- L25, Mon 10/29.* – [2, §4.3] Lebesgue Decomposition
- L26, Wed 10/31.* – [2, §3.5, §4.5] The dual of L^p .
- L27, Fri 11/2.* * The σ -finite case.
- Integration on Product Spaces.
- [2, §5.1] Product σ -algebras.
- L28, Mon 11/5.* – [2, §5.2] Product measures, and Tonelli's theorem.
- L29, Wed 11/7.* – [2, §5.2] Fubini's theorem.
- [2, §5.3] Distribution functions and applications.
- L30, Fri 11/9.* • Convolutions
- [5, §7] Young's inequality, Approximate identities.
- L31, Mon 11/12.* – [5, §7] L^p -convergence of approximate identities.
- Fourier Series.
- [6, §1] L^2 convergence, and Cesàro sums.
- L32, Wed 11/14.* – Riemann Lebesgue Lemma, and Sobolev inequalities.
- L33, Fri 11/16.* – Riemann Lebesgue Lemma, and Sobolev inequalities.
- L34, Mon 11/19.* • Differentiation
- [2, §6.1] Change of variable.
 - [2, §6.2; 5, §7] Vitali covering and the Maximal function.
 - [2, §6.2; 5, §7] Lebesgue points and differentiation of measures.
 - [2, §6.3; 5, §7] Fundamental theorem of Calculus.
- L35, Mon 11/26.* – [2, §6.2; 5, §7] Vitali covering and the Maximal function.
- L36, Wed 11/28.* – [2, §6.2; 5, §7] Lebesgue points and differentiation of measures.
- L37, Fri 11/30.* – [2, §6.3; 5, §7] Fundamental theorem of Calculus.
- L38, Mon 12/3.* • Fourier Transform
- [3, §8.3; 5, §9] L^1 theory and inversion.
 - [3, §8.3; 5, §9] Plancherel theorem and L^2 -theory.
 - [3, §9.3] Sobolev spaces and embeddings.
- L39, Wed 12/5.* – [3, §8.3; 5, §9] Plancherel theorem and L^2 -theory.
- L40, Fri 12/7.* – [3, §9.3] Sobolev spaces and embeddings.

References

- [1] R. G. Bartle, *The elements of integration and Lebesgue measure*, Wiley Classics Library, John Wiley & Sons Inc., New York, 1995. Containing a corrected reprint of the 1966 original [*The elements of integration*, Wiley, New York; MR0200398 (34 #293)]; A Wiley-Interscience Publication. MR1312157 (95k:28001)
- [2] D. L. Cohn, *Measure theory*, Birkhäuser Boston, Mass., 1980. MR578344 (81k:28001)
- [3] G. B. Folland, *Real analysis*, 2nd ed., Pure and Applied Mathematics (New York), John Wiley & Sons Inc., New York, 1999. Modern techniques and their applications; A Wiley-Interscience Publication. MR1681462 (2000c:00001)
- [4] H. L. Royden, *Real analysis*, 3rd ed., Macmillan Publishing Company, New York, 1988. MR1013117 (90g:00004)
- [5] W. Rudin, *Real and complex analysis*, 3rd ed., McGraw-Hill Book Co., New York, 1987. MR924157 (88k:00002)
- [6] W. Schlag, *Lecture notes on Harmonic Analysis*, available at <http://www.math.uchicago.edu/~schlag/harmonicnotes.old.pdf>.