

Stochastic Calculus Lecture schedule.

Gautam Iyer, Fall 2011

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| <p><i>L1, Mon 8/29.</i></p> <ul style="list-style-type: none"> • Stochastic processes <ul style="list-style-type: none"> – Basic definitions. (Filtrations, stopping times, etc.) – Exit times of continuous processes from domains are stopping times. | <p><i>L14, Wed 10/12.</i></p> <ul style="list-style-type: none"> * Statement of Itô's formula. * Lévy's criterion (sufficient part). * Characterization of the expected exit time of Brownian motion from a disk. * Proof of the Itô formula. * Stratonovich integrals. |
| <p><i>L2, Wed 8/31.</i></p> <ul style="list-style-type: none"> • Continuous time martingales <ul style="list-style-type: none"> – Doob's Martingale inequalities – Existence of RCLL modifications (no proof). – Submartingale convergence | <p><i>L15, Mon 10/17.</i></p> <ul style="list-style-type: none"> • Martingale representation theorem <ul style="list-style-type: none"> – Orthogonal decomposition of \mathcal{M}^2 into Itô integrals and their complement. – Martingale representation theorem for Brownian motion. – Itô representation theorem. |
| <p><i>L3, Wed 9/7.</i></p> <ul style="list-style-type: none"> – Optional sampling. – Local martingales. | <p><i>L16, Mon 10/24.</i></p> <ul style="list-style-type: none"> • Properties of Brownian Motion <ul style="list-style-type: none"> – Markov and Strong Markov properties. – Reflection principle – Computation of passage time densities – Blumenthal and Kolmogorov 0-1 laws. – Zero set of Brownian motion – Running maximum – Law of iterated logarithm |
| <p><i>L4, Fri 9/9.</i></p> <ul style="list-style-type: none"> – Completeness of \mathcal{M}^2, \mathcal{M}_c^2. – Quadratic variation. <ul style="list-style-type: none"> * Definition (as limit of sums of squares of increments), and proof of existence. | <p><i>L17, Wed 10/26.</i></p> <ul style="list-style-type: none"> • The Girsanov Theorem <ul style="list-style-type: none"> – Statement and proof. – Passage times of Brownian motion with a drift. – Regularity of exponential martingales. |
| <p><i>L5, Mon 9/12.</i></p> <ul style="list-style-type: none"> * $\langle M \rangle$ is the unique continuous, adapted, increasing process such that $M^2 - \langle M \rangle \in \mathcal{M}_{c,loc}$ for all $M \in \mathcal{M}_{c,loc}$. * Joint quadratic variation. | <p><i>L18, Mon 10/31.</i></p> <ul style="list-style-type: none"> • Stochastic Differential equations <ul style="list-style-type: none"> – Strong solutions. <ul style="list-style-type: none"> * Existence and uniqueness. – Weak solutions. <ul style="list-style-type: none"> * Tanaka's example. * Existence and uniqueness via the Girsanov theorem. |
| <p><i>L6, Wed 9/14.</i></p> <ul style="list-style-type: none"> • Brownian Motion <ul style="list-style-type: none"> – Construction of Brownian Motion <ul style="list-style-type: none"> * Kolmogorov Čentsov theorem | <p><i>L19, Wed 11/2.</i></p> <ul style="list-style-type: none"> – Diffusions <ul style="list-style-type: none"> – Markov and Strong Markov properties. – Dynkin's formula, generators. – Recurrence of Brownian Motion. – Kolmogorov backward equation. |
| <p><i>L7, Mon 9/19.</i></p> <ul style="list-style-type: none"> – (Dima) Construction of Brownian Motion via Fourier Series. | <p><i>L20, Mon 11/7.</i></p> <ul style="list-style-type: none"> – Feynman-Kac formula. – Kolmogorov forward equation. – Dirichlet-Poisson problems in bounded domains. |
| <p><i>L8, Wed 9/21.</i></p> <ul style="list-style-type: none"> – (Dima) Hölder properties of trajectories. | |
| <p><i>L9, Mon 9/26.</i></p> <ul style="list-style-type: none"> – (Dima) Right continuity of the Brownian Filtration – (Dima) Blumental's 0 – 1 law. | |
| <p><i>L10, Wed 9/28.</i></p> <ul style="list-style-type: none"> – (Dima) Lévy's criterion (necessary part). – (Dima) Strong Markov property (statement only). | |
| <p><i>L11, Mon 10/03.</i></p> <ul style="list-style-type: none"> • Stochastic Integration <ul style="list-style-type: none"> – Construction of the Itô integral. <ul style="list-style-type: none"> * Itô isometry. | |
| <p><i>L12, Wed 10/05.</i></p> <ul style="list-style-type: none"> * Approximation by simple functions. [Only proved \mathcal{L}_0 is dense in $\mathcal{L}(M)$, when $\langle M \rangle$ is absolutely continuous.] * Joint quadratic variation of Itô integrals * Martingale characterization. | |
| <p><i>L13, Mon 10/10.</i></p> <ul style="list-style-type: none"> * Integration with respect to local martingales. | |