Math 269 Syllabus and Lecture Schedule.

Gautam Iyer, Spring 2012

L1, Wed 1/18.
- *(Sec. 1.4).* Reminder about dot and cross products in $\mathbb{R}^n$.
  - Distances, length, and angles.
  - Cauchy Schwarz and triangle inequalities.

L2, Fri 1/20.
- *(Sec. 1.5).* Topology of $\mathbb{R}^n$.
  - Open and closed sets.

L3, Mon 1/23.
  - Interior, exterior, boundary.
- Sequences and convergence.
  - Convergence of all coordinates $\iff$ convergence.
  - Uniqueness of limits.

L4, Wed 1/25.
  - Limits of sums, scalar products of sequences.

L5, Fri 1/27.
- Limits of functions and continuity.
  - Continuity of composition

L6, Mon 1/30.
  - Sums, products, rational functions.

L7, Wed 2/01.
  - Iterated limits (not in the book)
    - $\lim_{x\to a}\lim_{y\to b} f(x,y)$ need not equal $\lim_{x\to a}\lim_{y\to b} f(x,y)$.
    - Even if iterated limits both exist and are equal, the full limit $\lim_{(x,y)\to(a,b)} f(x,y)$ need not exist.
    - Even if the full limit exists, iterated limits need not exist (or even be defined).
    - If, however, the full limit exists, and $\lim_{x\to a} f(x,y)$ exists for all y close enough to b, then the iterated limit $\lim_{y\to b}\lim_{x\to a} f(x,y)$ exists and equals the full limit.
    - Consequently, if the full limit exists, and both iterated limits make sense, the two iterated limits must be equal.

L8, Fri 2/03.
  - Series of vectors and matrices.
    - Reminder of convergence tests.
    - Cauchy sequences. Absolute convergence implies convergence.

L9, Mon 02/06.
    - $|A| < 1 \implies \sum_0^\infty A^n = (I-A)^{-1}$.
    - The set of all invertible matrices is open.
- *(Sec. 1.6).* Compactness

L10, Wed 02/08.
  - Definitions. Compact implies closed and bounded.
  - Sup and Inf

L11, Fri 02/10.
  - Monotone bounded sequences are convergent.
  - Bolzano Weirstrauss: Any bounded sequence in $\mathbb{R}$ has a convergent subsequence.

L12, Mon 02/13.
  - Closed and bounded in $\mathbb{R}^n$ implies compact.
  - A continuous image of a compact set is compact.
  - Continuous functions on compact sets attain their bounds. (Extreme value theorem.)

L13, Wed 02/15.
- Derivatives.
  - Product, quotient, chain rules, etc.
  - Derivatives at maxima.

L14, Fri 02/17.
- Midterm.

L15, Mon 02/20.
- *(Sec. 1.6).* (Lagrange) Mean value theorem
  - Applications (e.g. increasing functions).

L16, Wed 02/22.
  - Cauchy mean value theorem (not in the book)
  - L'Hospitals rule.
- *(Sec. 1.7).* Derivatives of functions of more than one variable.
  - Partial derivatives

L17, Fri 02/24.
  - Directional derivatives
  - (Full) derivative, and Jacobian

L18, Mon 02/27.
  - E.g. where the Jacobian is not the derivative.
  - Extension to $f : \mathbb{R}^m \to \mathbb{R}^n$.

L19, Wed 02/29.
  - *(Sec. 1.8).* $D(fg)_a(h) = (Df_a)(h)g(a) + f(a)(Dg_a(h))$.
    - f differentiable iff $f(a+h) - f(a) = Df_a(h) + o(|h|)$.
    - Proof of product rule.

L20, Fri 03/02.
  - Chain rule, and proof.

L21, Mon 03/05.
  - Examples and applications of the chain rule.

L22, Wed 03/07.
  - Equality of mixed partials.

L23, Mon 03/19.
  - Mean value theorem

L24, Wed 03/21.
  - Taylors theorem.

L25, Fri 3/23.
- Midterm.

L26, Mon 03/26.
- Maxima / minima
  - Negative definite quadratic forms.

L27, Wed 03/28.
  - Sufficient criteria for local maxima.

L28, Fri 03/30.
- Inverse function theorem.
  - Assume $Df_a = I$ for simplicity.
  - Step 1: Injectivity.

  * Put $\varphi(x) = f(x) - x$.
  * Arrange $|Df_x| < \frac{1}{2n}$ for x close to a.
  * Use the mean value theorem to show $|f(x)-f(x')| \geqslant \frac{1}{2}|x-x'|$ for x, x' close to a.
- Step 2: Surjectivity.
  * Put $\varphi(x) = |f(x) - y|^2$.
  * Arrange for φ to have an interior maximum.
  * At an interior maximum, $D\varphi_x = 0$, forcing $f(x) = y$.

L29, Mon 04/02.
- Step 3: Differentiability of the inverse.
  * Let $x, x_0 \in U_1$, $y = f(x)$, $y_0 = f(x_0)$, $S = Df_{x_0}$, $T = S^{-1}$.
  * $g(y) - g(y_0) - T(y - y_0) = x - x_0 - T(f(x) - f(x_0)) = TF(x - x_0)$, where $F(x - x_0) = f(x) - f(x_0) - S(x - x_0)$.
  * Differentiability of f and the inequality $|f(x) - f(x_0)| \geqslant \frac{1}{2}|x - x_0|$ finishes the proof.

L30, Wed 04/04.
- *(Sec. 2.10).* Implicit function theorem.

L31, Fri 04/06.
  - Manifolds, regular values, etc.

L32, Mon 04/09.
- *(Sec. 3.7).* Lagrange multipliers
  - Motivation, statement (and one example)

L33, Wed 04/11.
  - Proof.

L34, Fri 04/13.
- Integration
  - Double sums. Motivation of multiple integrals.
  - Mass, volume, etc.
  - Computing integrals: One variable – Fundamental Theorem of Calculus. Multi variable – Reduce to an iterated integral.
  - Fubini's theorem

L35, Mon 04/16.
  - Examples and counter examples.
- Change of variables.
  - Polar coordinates.
  - Motivation: Determinants measure volumes of parallelepipeds.

L36, Wed 04/18.
  - E.g Computing $\int_{-\infty}^{\infty} e^{-x^2}\,dx$.
- Volume integrals.
  - Fubini's theorem and change of variables.
  - Spherical coordinates.

L37, Mon 04/23.
  - Volumes of spheres.
- Line integrals.
  - Motivation and examples (e.g. work done).
  - Parametrization invariance.

L38, Wed 04/25.
- Green's theorem
  - Proof on a square.
  - Proof on a C^2 image of a square.

L39, Fri 04/27.
- Surface integrals
  - Motivation, definition
  - Form notation.

L40, Mon 04/30.
- Stokes theorem
  - Proof for C^2 images of a square.

L41, Wed 05/02.
- Divergence theorem
  - Proof on a cube & applications.

L42, Fri 05/04.
- Winding number and fundamental theorem of algebra.