Math 880 Midterm.

October 13, 2010

*This is a closed book test. You have 90 minutes. You may (or may not) find the questions getting progressively harder. You may use without proof any result from class and homework, unless **any** of the following hold:*

- *You are explicitly instructed otherwise.*
- *The proof of the result you want to use relies on the question you're asked.*
- *The result you want to use is the question you're asked to solve.*
- *The result you want to use has not (yet) been proved in class.*

In this exam, Ω always denotes a probability space, with measure P. Brownian motion will usually be denoted by W or B, and the underlying filtration (if not explicitly mentioned) is denoted by $\mathcal{F} = \{\mathcal{F}_t\}_{t \geqslant 0}$, and is always assumed to satisfy the usual conditions.

1. Let τ be a stopping time. Are the random times $\frac{\tau}{2}$ and $\frac{3\tau}{2}$ necessarily stopping times? Prove, or provide a counter example.
2. Suppose X is process such that $E|X_t - X_s| \leqslant |t-s|^3$. Must X have a constant in time modification? Prove, or provide a counter example.
3. Let B be a standard 1D Brownian starting at 0, and let $\tau = \sup\{t \leqslant 1 \mid B_t = 0\}$. Is τ a stopping time? Prove or disprove.
4. Let $0 \leqslant s < t$. Show that (almost surely) Brownian motion is not monotone on the time interval (s,t).
5. Let B be a (standard 1D) Brownian motion and τ a stopping time. Define the process X by $X_t = 2B_{\tau \wedge t} - B_t$. Compute $\varphi(\xi, t) = E^0 e^{i\xi X_t}$ explicitly as a function of ξ and t.