Stochastic Calculus Lecture schedule.

Gautam Iyer, Fall 2010

L1, Mon 8/23.
- Stochastic processes
  - Basic definitions. (Filtrations, stopping times, etc.)
  - Exit times of continuous processes from domains are stopping times.

L2, Wed 8/25.
- Continuous time martingales
  - Doob's Martingale inequalities
  - Existence of RCLL modifications (no proof).
  - Submartingale convergence

L3, Mon 8/30.
  - Optional sampling.
  - Local martingales.

L4, Wed 9/1.
  - Completeness of $\mathcal{M}^2$, $\mathcal{M}_c^2$.
  - Quadratic variation.
    * Definition and proof of existence

L5, Wed 9/8.
    * $M^2 - \langle M \rangle \in \mathcal{M}_{c,\mathrm{loc}}$ for all $M \in \mathcal{M}_{c,\mathrm{loc}}$.
    * Joint quadratic variation.

L6, Fri 9/10.
- Brownian Motion
  - Construction of Brownian Motion
    * Daniel Kolmogorov consistency theorem

L7, Mon 9/13.
    * Kolmogorov Čentsov theorem
    * Brownian families, Wiener Space, Wiener measure

L8, Wed 9/15.
  - Markov processes and families
    * Markov property of Brownian Motion
    * Equivalent formulations of the Markov property

L9, Mon 9/27.
    * Time shifts
  - Strong Markov processes and families
    * Strong Markov property for Brownian motion
    * Restarting Brownian motion at stopping times

L10, Wed 9/29.
    * Computation of passage time densities
    * Blumenthal and Kolmogorov 0-1 laws.

L11, Mon 10/4.
  - Sample path properties
    * Zero set of Brownian motion
    * The running maximum of Brownian motion
    * Law of iterated logarithm

L12, Wed 10/6.
  - Weak convergence and Donsker's invariance principle. (Omitted proof of tightness)

L13, Mon 10/11.
- Stochastic Integration
  - Construction of the Itô integral.
    * Itô isometry.
    * Elementary properties.

L14, Mon 10/18.
    * Approximation by simple functions. (Omitted the proof when $\langle M \rangle$ is not absolutely continuous)
    * Kunita Watanabi inequality.

L15, Wed 10/20.
    * Joint quadratic variation of Itô integrals
    * Martingale characterization.
    * Integration with respect to local martingales.

L16, Mon 10/25.
  - The Itô formula.

L17, Wed 10/27.
  - Stratonovich integrals.
  - Lévy's characterization of Brownian Motion.

L18, Mon 11/1.	• Martingale representation theorem.
L19, Wed 11/3.	• The Girsanov Theorem
	– Statement and proof.
L20, Mon 11/8.	– Passage times of Brownian motion with a drift.
	– Regularity of exponential martingales.
L21, Wed 11/10.	• Stochastic Differential equations
	– Strong solutions.
	∗ Existence and uniqueness.
L22, Mon 11/15.	– Weak solutions.
	∗ Tanaka's example.
	∗ Existence.
L23, Wed 11/17.	• Diffusions
	– Markov and Strong Markov properties.
L24, Mon 11/22.	– Dynkin's formula, generators.
	– Recurrence of Brownian Motion.
L25, Mon 11/29.	– Kolmogorov backward equations
	– Time inhomogeneous diffusions
L26, Wed 12/1.	– Kolmogorov forward equations.
	– Feynman Kac formula.
L27, Fri 12/3.	– The Dirichlet and Poisson problems in bounded domains.