21341 Linear Algebra: Midterm 1.

Wed 03/23

- *This is a closed book test. No calculators or computational aids are allowed.*
- *You have 50 minutes. The exam has a total of 5 questions and 50 points.*
- *You may use without proof any result that has been proved in class or on the homework, provided you* ***CLEARLY*** *state the result you are using.*
- *Difficulty wise: #1 $\leqslant$ #2 $\approx$ #3 $<$ #4 $<$ #5. The first inequality is not strict. The last two are. (The last inequality is slightly 'stricter' than the others.)*

In this exam, we always assume V is a vector space over a field F.

$\boxed{10}$ 1. Let $F = \mathbb{R}$, and V be the vector space consisting of all real valued functions with domain $[0, 1]$. For any $f \in V$, define $Tf = f(1) - f(0) + 1$. Does $T \in \mathcal{L}(V, \mathbb{R})$ (i.e. is T a linear transformation)? Prove / disprove.

$\boxed{10}$ 2. Let $M = \left(\begin{smallmatrix} -1 & 1 & 1 \\ 1 & -1 & 2 \end{smallmatrix}\right) \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$. Find a basis of $\ker(M)$. [Recall $\ker(M) = \{x \in \mathbb{R}^3 \mid Mx = 0\}$.]

$\boxed{10}$ 3. Does there exist $T \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^3)$ such that $\operatorname{im}(T) = \ker(T)$? If yes, find an example. If no, prove it. [If you say 'Yes' and produce an example, then you should also prove that your example has the desired properties.]

$\boxed{10}$ 4. Suppose $T \in \mathcal{L}(V, V)$ is an injective linear transformation such that $T^2 = 4T$. Find all eigenvalues of T. (You should prove whatever answer you get.)

$\boxed{10}$ 5. Let $U \subseteq F^n$ be a subspace with $\dim(U) = m < n$. Show that there exists an $(n - m) \times n$ dimensional matrix M such that $U = \ker(M)$.