

# Stochastic Calculus for Finance I: Midterm.

2019-02-07

- *This is a closed book test. No electronic devices may be used. You may not give or receive assistance.*
- *You have 90 minutes. The exam has a total of 5 questions and 25 points.*
- *The questions are roughly ordered by difficulty. Good luck and 新年快乐.*

In this exam  $W$  always denotes a standard Brownian motion, and the filtration  $\{\mathcal{F}_t | t \geq 0\}$  (if not otherwise specified) is the Brownian filtration.

5. Define the process  $X$  by  $X(t) = \int_0^t W(s)^2 dW(s)$ . Compute  $\mathbf{E}X(t)$  and  $\mathbf{E}[X(t)^2]$ . Express your answers as explicit functions of  $t$  without involving  $W$ , expectations or integrals.
5. Let  $X(t) = te^{-2W(t)}$ . Find a martingale,  $M$ , and an adapted process with finite first variation,  $B$ , such that  $X(t) = M(t) + B(t)$ . (You may leave your answers as a combination of Riemann and or Itô integrals.)
5. Let  $X(t) = \exp(-tW(t)^2) + \int_0^t W(s)^2 ds - \int_0^t \exp(W(s)) dW(s)$ . Compute  $[X, X](t)$ . (You may leave your answer as an integral.)
5. Let  $X$  and  $Y$  be two independent standard normal random variables. Let  $Z = X + Y$ , and find  $\mathbf{E}(X | Z)$ . Your final answer may involve  $X$  and  $Y$  but should not involve any expectations or integrals.
5. Let  $M(t) = \int_0^t rW(r) dW(r)$ . For  $0 \leq s < t$ , compute  $\mathbf{E}(M(t)^4 | \mathcal{F}_s)$ .