In this exam $W$ always denotes a standard Brownian motion, and the filtration $\{\mathcal{F}_t\mid t \geq 0\}$ (if not otherwise specified) is the Brownian filtration.

1. Let $X(t) = (t + 2W(t))^3$. Explicitly find adapted processes $b, \sigma$ such that

$$X(t) = X(0) + \int_0^t b(r) \, dr + \int_0^t \sigma(r) \, dW(r).$$

2. Let $s < t$ and let $X = W(s) + W(t)$. What is the distribution of $X$? Also compute the mean and variance of $X$.

[You should explain why the distribution of $X$ is what you say it is, and not simply state it. Moreover you should express the mean and variance of $X$ only in terms of $s$ and $t$ without involving expected values or integrals.]

3. Let $\lambda > 0$ and define $X(t) = \exp(\lambda W(t))$. Find the quadratic variation of the process $X$. Express your answer in the form $\int_0^t f(\lambda, s, W(s)) \, ds$ for some function $f$ which you explicitly compute the formula for.

4. Let $X, Y$ be two independent random variables such that $|X| \leq 944$ and $Y$ is a standard normal. Compute $E(X^2e^{XY} \mid X)$. Your final answer can involve $X$, but should not involve any expectations or integrals.

5. Given $\lambda > 0$, compute

$$E\left(e^{-\lambda W(t)} \int_0^t e^{\lambda W(s)} \, dW(s)\right).$$

Express your final answer as a function of $t$ and $\lambda$ without involving expected values or integrals.