Stochastic Calculus for Finance I: Midterm.

2017-11-20.

- *This is a closed book test. No electronic devices may be used. You may not give or receive assistance.*
- *You have 90 minutes. The exam has a total of 5 questions and 25 points.*
- *The questions are roughly ordered by difficulty. Good luck* ☺.

In this exam W always denotes a standard Brownian motion, and the filtration $\{\mathcal{F}_t \mid t \geqslant 0\}$ (if not otherwise specified) is the Brownian filtration.

$\boxed{5}$ 1. Let $X(t) = (t + 2W(t))^3$. Explicitly find adapted processes b, σ such that

$$X(t) = X(0) + \int_0^t b(r)\,dr + \int_0^t \sigma(r)\,dW(r)\,.$$

$\boxed{5}$ 2. Let $s < t$ and let $X = W(s) + W(t)$. What is the distribution of X? Also compute the mean and variance of X.

[You should explain why the distribution of X is what you say it is, and not simply state it. Moreover you should express the mean and variance of X only in terms of s and t without involving expected values or integrals.]

$\boxed{5}$ 3. Let $\lambda > 0$ and define $X(t) = \exp(\lambda W(t))$. Find the quadratic variation of the process X. Express your answer in the form $\int_0^t f(\lambda, s, W(s))\,ds$ for some function f which you explicitly compute the formula for.

$\boxed{5}$ 4. Let X, Y be two *independent* random variables such that $|X| \leqslant 944$ and Y is a standard normal. Compute $\boldsymbol{E}(X^2 e^{XY} \mid X)$. Your final answer can involve X, but should not involve any expectations or integrals.

$\boxed{5}$ 5. Given $\lambda > 0$, compute

$$\boldsymbol{E}\Big(e^{-\lambda W(t)} \int_0^t e^{\lambda W(s)}\,dW(s)\Big)\,.$$

Express your final answer as a function of t and λ without involving expected values or integrals.