21-269 Vector Analysis: Midterm 1.

2017-02-22

- *This is a closed book test. No electronic devices may be used. You may not give or receive assistance.*
- *You have 50 minutes. The exam has a total of 5 questions and $40\varepsilon - 1$ points.*
- *You may use any result proved in class or any regular homework problem* ***PROVIDED*** *it is* independent *of the problem you want to use the result in. (You must also* ***CLEARLY*** *state the result you are using.)*
- *The first question is evil. The remaining are roughly in increasing order of difficulty. Good luck* ⌣̈.

A wrong or blank answer on the first question is worth -1 points. The points on the remaining questions are multiples of ε, where $\varepsilon > 0$ is a small number that will be chosen later to ensure the “curve” is a nice bell curve with mean 0, as drawn in class.

1. What is $\pi^e - e^\pi$ to 12 significant digits? (No electronic devices 😱😈!)

2. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x) = |x|$.

   $\boxed{5\varepsilon}$ (a) Find the derivative of f at the point $(3, 4)$. Explain why f is differentiable at this point.

   $\boxed{5\varepsilon}$ (b) Is f differentiable at 0? Prove your answer.

3. $\boxed{10\varepsilon}$ Suppose $a \in \mathbb{R}^2$, $\alpha, \beta \in \mathbb{R}^3$ and $f, g\colon \mathbb{R}^2 \to \mathbb{R}^3$ are such that $\lim_{x \to a} f(x) = \alpha$ and $\lim_{x \to a} g(x) = \beta$. Show directly using the ε-δ definition that $\lim_{x \to a} (2f(x) - 3g(x))$ exists.

4. $\boxed{10\varepsilon}$ True or false:

   If $f\colon (0,1) \to \mathbb{R}$ is uniformly continuous, then the sequence $(f(\frac{1}{n}))$ is convergent.

   Prove it, or find a counter example.

5. $\boxed{10\varepsilon}$ True or false:

   If $f\colon [0,1] \to [0,1]$ is continuous, then there exists $x \in [0,1]$ such that $f(x) = x$.

   Prove it, or find a counter example.