21-269 Vector Analysis: Midterm 1.

Feb 17[th], 2012

- *This is a closed book test. No calculators or computational aids are allowed.*
- *You have 50 mins. The exam has a total of 5 questions and 50 points.*
- *You may use without proof any result that has been proved in class or on the homework, provided you **CLEARLY** state the result you are using.*
- *Difficulty wise: #1 $\leqslant$ #2 $\leqslant$ #3 $\leqslant$ #4 $<$ #5. The last inequality is strict.*

[10] 1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \frac{x_1^2}{x_1+x_2} & \text{if } x_1 + x_2 \neq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Does $\lim_{x\to 0} f(x)$ exist? Prove your answer. [Here $x = \left(\begin{smallmatrix} x_1 \\ x_2 \end{smallmatrix}\right) \in \mathbb{R}^2$.]

[10] 2. Let S, T be two non-empty subsets of $\mathbb{R}$, such that $\forall s \in S, t \in T$ we have $s \leqslant t$. Show that $\sup(S) \leqslant \inf(T)$.

[10] 3. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \sqrt{|x|}$. Give a direct ε-δ proof that $\lim_{x\to 1} f(x) = 1$.

[10] 4. Suppose $U \subseteq \mathbb{R}^2$ is open. For a given $x_0 \in \mathbb{R}$, define $V \subseteq \mathbb{R}$ by

$$V = \{y \in \mathbb{R} \bigm| (x_0, y) \in U\}.$$

Show that $V \subseteq \mathbb{R}$ is open.

[10] 5. If $S \subseteq \mathbb{R}^2$ is *both* open and closed show that $S = \mathbb{R}^2$ or $S = \emptyset$. [You may use the one dimensional version you proved on your homework.]