1. Find the full Fourier series of the function \( f(x) = x \) on the interval \((-\pi, \pi)\).

2. For a general function \( f : [-\pi, \pi] \to \mathbb{R} \), find \( \int_{-\pi}^{\pi} f(x)^2 \, dx \) in terms of the coefficients of its full Fourier series. Use this to compute \( \sum_{n=1}^{\infty} \frac{1}{n^2} \). [This was HW5, Q2(a).]

3. Let \( D = \{ x \in \mathbb{R}^2 \mid |x| \leq a \} \) be a disk of radius \( a \). Let \( u \) be a function such that \( \Delta u = 0 \) on the interior of \( D \), and \( u(x) \geq 0 \) for all \( x \in D \). If \( 0 < r < a \) and \( |x| \leq r \), show that
   \[
   u(x) \leq \left( \frac{a + r}{a - r} \right) u(0).
   \]
   [HINT: Use the Poisson formula; this is the Harnack inequality and was on HW6 Q5]

4. Suppose \( u \) solves the heat equation \( \partial_t u - \kappa \partial_x^2 u = 0 \) for \( x \in (0, \pi) \) and \( t > 0 \) with Dirichlet boundary conditions \( u(0, t) = u(\pi, t) = 0 \) and initial data \( u(x, 0) = f(x) \). You may assume \( \kappa > 0 \) and \( \int_0^{\pi} |f(x)|^2 \, dx < \infty \). True or false: Does there exist \( \alpha > 0 \) such that
   \[
   \lim_{t \to \infty} e^{\alpha t} \int_0^{\pi} u(x, t)^2 \, dx = 0?
   \]
   Prove it, or find a counter example. [Hint: Separate variables, and use results about the Fourier series. We’ve seen a similar result on the homework before, however, the convergence required in this question is stronger.]